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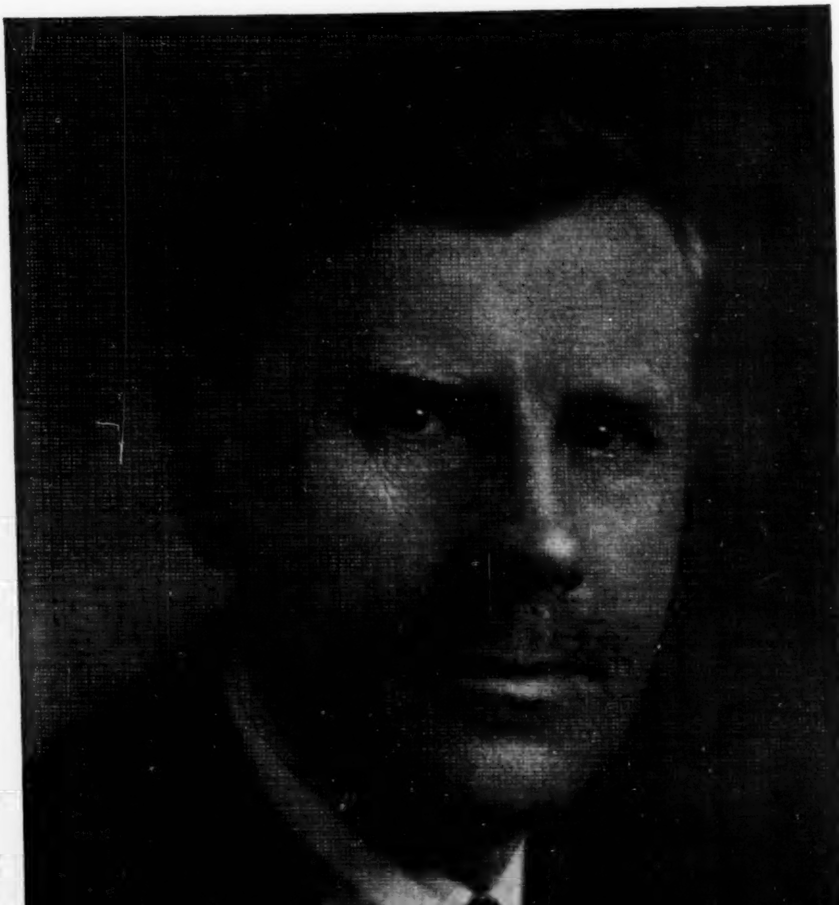

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THE MATHEMATICS TEACHER

Volume XXXII Number 5

Edited by William David Reeve

New Trends in the Teaching of Mathematics

By John W. Studebaker
United States Commissioner of Education, Washington, D. C.

Introduction

In a few, short months another decade will draw to its close. It is a fitting time, therefore, to consider what significant changes the past ten years have brought in the teaching of mathematics.

In discussing trends in mathematics, in the United States alone, I might go back two centuries or more but the topic assigned to me suggests that my discussion be limited to the immediate past, the present, and the future. My own experience as a teacher, supervisor, and author in the field of mathematics covers the greater part of the past two and a half decades. It is of this period that I am best prepared to speak, and it is within this period that the beginnings of present trends emerged.

Perhaps the best way to give a proper perspective to present and probable future trends in mathematics is to present brief characterizations, decade by decade, of the major issues which have held the attention of educators during this century, and then to examine the import of these issues for the future.

The Decade, 1900 to 1910

At the turn of the century there was increasing attention to the content of elementary school arithmetic. For many years educators had been suggesting that our courses of study were overloaded with such antiquated topics as cube root, partial payments, obsolete and obsolescent weights and measures, and involved common fractions. The inrush of a large number of school subjects resulted in a strenuous competition for the hours of the school day. Arithmetic with its overload of socially nonsignificant materials naturally was forced to retreat both in its time allotment and in its content, an outcome with which I think most of us were in agreement.

The Decade, 1910 to 1920

The movements in mathematics during the period from 1910 to 1920 were profoundly influenced by the development of a scientific educational psychology and its applications to the teaching of the elementary school subjects. In 1913–1914 Thorndike published his classic three volumes on educational psychology, Volume II of which brought together for the first time in English practically all of the literature on learning and the principles of distributed practice. At about the same time many of Thorndike's students, Hahn, Kirby, and others, were making detailed studies of learning in arithmetic with special reference to the amounts and distribution of drill, a development which extends even to the present time. Even earlier

were the efforts of Stone and Woody to construct tests that would measure the results in the teaching of arithmetic. Under the stimulus of these beginnings in the scientific control and measurement of learning in arithmetic, Courtis, the speaker, and others, developed extensive systems of arithmetic drill designed for the individualization and specialization of practice in this subject.

Toward the end of this decade, Thorndike published a series of textbooks in arithmetic in which he attempted to apply his psychology of learning in a concrete way, particularly with reference to the management of drill. Prior to this time, textbooks had been based upon the theory of initial over-learning of the skills by piling up enormous amounts of practice at the time of first-learning in the attempt to negate the forgetting through disuse when the pupil moved from a given topic to the next. At this period of our thinking the classroom teacher was tremendously impressed by the low level of retention of information and skill after relatively short intervals of time. Teachers almost without exception justified the poor accomplishments of their pupils with the excuse that "they came to me poorly prepared," an attitude which was at least a half-truth. It required some time to dig to the real root of the matter, which was that *forgetting is inevitable unless combatted by the provision of drill on former operations with regularly spaced drill for maintenance of skills.* This conclusion, known for more than a half century in the experimental psychology laboratories since the time of Wundt and his students, now began to show its influence on the course of study and textbook in arithmetic. In the future wasteful and inefficient initial over-learning was to be replaced by what has been called the theory of "small doses frequently"; in a word, by distributed practice for the maintenance of and gradual growth in facility in the fundamental processes—a movement which was to replace the previous attempts at holding skills at a functional level by means of the so-called cyclic curriculum in which school subjects or topics were repeated in cycles of two or three years. This, then, was the status of the theory of drill in the year 1920.

There was a second general movement of vast significance during the decade 1910–1920, viz., the emergence of the *doctrine of social utility.* Wilson, Woody, and a number of others, were examining the actual practices of adults in the use of mathematics in the home and in business, particularly the latter. In 1919 Wilson published his "Survey of the Social and Bussines Usage of Arthmetic" as Volume 100 of the *Teachers College Contributions to Education,* which presented findings to the effect that the average person uses little or no arithmetic beyond the four fundamental processes, a few fractions, and United States money. Studies of this sort have continued to appear and, collectively, have exerted marked influence in further eliminations of useless topics as well as the abandonment of many of the harder aspects of topics which are still retained. The effect of these studies has not been as great as these investigators had hoped partly because of educational inertia and partly because many educators feel that these studies do not rest upon a broad enough base. To be specific, the attempts to ascertain the socially functioning topics in arithmetic by the method of surveys of usage are as yet too limited in scope. Certain kinds of practices, such as the arithmetic of sales slips, seem to have been adequately sampled; on the other hand, whole areas are as yet untouched. We know little or nothing of the mathematics of the farm, trades and industries, and numerous other areas. In fact, the fields surveyed to date probably do not represent more than ten per cent of the areas which must be inventoried before a mathematics curriculum can be set down in terms of social usage.

The Decade, 1920 to 1930

The activities of the years from 1920 to 1930 are difficult to characterize unless a very broad descriptive phrasing is employed. If we examine the developments microscopically, a bewildering current of movements may be discerned. This period was one of crystallization and consolidation of gains along a variety of fronts. Perhaps the best characterization, from a broad point of view, is to describe the period as one devoted to the application and extension of recently developed theories of learning—a view necessitating the discussion of a number of tendencies which may not appear to be closely interrelated at first glance.

1. *The Analysis of Errors*—One of the currents of this decade was the controversy over the specificity of errors. A school of thought, headed by Garry C. Myers, produced evidence that certain types of errors in arithmetic computation are of fairly constant or stable character. Not all authorities subscribed to this view, nor is the evidence entirely unambiguous. However, there can be no doubt that many types of errors may rightfully be regarded as constant in contrast with the transitory difficulties which always arise in the early stages of learning. The significance of constant errors is evidence that it is probably sufficient to point out that teachers and textbook writers have been alert to the possibilities of using these findings to strengthen numerous weak spots in the pattern of arithmetic instruction.

2. *The Rise of the Concept of Diagnosis*—The work on specific errors in arithmetic mastery lead the thinkers in this field to the borrowing of methodology and terminology from the field of medicine. Diagnostic tests, often mislabelled and unreliable, made their appearance in vast numbers both as integral parts of textbooks and as independent tools for general application. The wholesale administration of such so-called "diagnostic" measures unquestionably resulted in a great deal of good. It directed the attention of teachers to many weaknesses in instruction and, in general, contributed a concreteness to their professional point of view. Another result was the gradual emergence of the conception that the idea of drill must be analyzed into a number of more specific varieties, each to be related to a different objective.

To be specific, at least four kinds of drill must be recognized by function. First, there is the drill that accompanies initial learning in order to "stamp in" the new skill, to use the phraseology of the stimulus-response school of psychology. Secondly, there is the drill or testing for the diagnosis of weaknesses in initial learning. This leads to reteaching of the specific elements which have not been mastered and to the third type of drill, commonly called "remedial" drill. Lastly, we may mention again the drill for maintenance and increased mastery of skills over the subsequent school years.

This highly analytic view of drill, although excellent from a purely logical point of view, and of great value to our thinking at the time, has resulted in the inevitable attack on its proponents. It was inherent in the situation that, sooner or later, critics would arise to say that this elaborate fabric of drilling, testing, diagnosing and reteaching meant just one thing—the teaching job was inadequate at the outset. The old saw that "an ounce of prevention is worth a pound of cure" was revived, and to good effect as may now be shown.

3. *The Textbook as a Learning Instrument*—The effect that I have just mentioned is, perhaps, most clearly evidenced by the change in textbooks between 1920 and 1930. Before this period the arithmetic textbook had followed a conventional pattern in introducing a pupil to a new operation. At the top of the page was a worked-out solution of a typical example, together with a few lines of explanation in precise adult mathematical language. To make matters worse, this explanation was usually set in smaller

type to still further diminish its usefulness to the child. With this lick and promise the pupil was directed to solve the problems on the page. Next day the process was repeated with a new skill, and so on. The best analogy to this situation which has come to my attention is the story of the poultryman, who dissatisfied with the size of eggs his hens were laying, secured an ostrich egg which he placed on a shelf in the henhouse with the slogan, "Look at this and do your best!"

With such a textbook at hand the teacher, of necessity, did most of the basic teaching at the blackboard, using the adopted text as a source of drill only. Like the other matters which we have been discussing, the reaction soon came. Textbooks appeared which attempted to lead the child, step by step, through the maze of difficulties attendant upon the first study of any new operation. One sample solution gave way to the use of many. Brief mathematical explanations yielded to detailed helps on every significant difficulty with the language in terms of the child's vocabulary. Only gradually, with many helps, was the pupil led to the stage of independent work. To a considerable degree the learning became an individual matter, with the teacher conserving her time and efforts for the assistance of the slower pupil. Habits of independent work and initiative were made possible to many pupils—a result of incalculable value in the general development of children.

Textbooks during this period showed many other significant changes in character, one of which may be given brief notice. As was to be expected, the testing movement exerted its influence within the textbook. Textbook authors soon hit upon the idea that certain kinds of drill work, particularly drill of the maintenance type, could serve many purposes simultaneously. By standardizing these periodic maintenance drills they could be made to supply a continuing inventory, or measurement, of progress without militating in the least against their effectiveness as pure drill. Because of the constant recurrence of the same types of examples and problems, pupils could discover the sources of their persistent errors and thus perform the function of self-diagnosis—a process which was both economical and valuable for its by-product, the development of independent habits of self-criticism and self-improvement. The provision of periodic standardized tests as an integral part of the text had yet a further value; it enabled the child to measure himself continuously. The child could plot his progress graphically and compete with his past record instead of being forced to seek what little satisfaction and motivation he could find in the conventional textbook directions, "See how many of these examples you can do in five minutes."

Time has permitted the mention of but a few of the many types of thinking that characterized the period of the "twenties" in mathematics. As I have said, this period was one of consolidating gains along many fronts. The textbook has been given considerable prominence as evidence of the trends of the decade, not because of any idea of glorifying the much abused textbook, but because the textbook is a barometer of thought, to a large degree, at all times. It is probably true that the course of study and the textbook first became markedly aware of the contributions of scientific education during the decade under discussion.

The Decade, 1930 to 1940

The temper of the decade that is now drawing to a close has been that of widespread challenging of present school practices. The progressive education movement has begun to be felt in the schools generally and there is a growing tendency to organize instruction in activity units instead of the traditional formal classes. So far as mathematics is concerned, the broad currents of the thinking of the decade may be exemplified by five questions which have been repeatedly asked by thoughtful educators.

1. *Does the activity program demand that formal classes in arithmetic be abandoned in favor of generalized activity units?*

The first impulse of the progressive education movement was the complete reorganization of the elementary school curriculum around activity centers thought to be of large social significance. Discrete subjects were abandoned and each activity was planned so as to draw its materials from any subject matter field. In doing this no distinction was made between the tool subjects, the informational subjects, the appreciational subjects, and so on.

A few years of experience with such a curricular organization revealed a weakness so far as the tool subjects are concerned. Thus we find Supt. Carleton Washburne of the Winnetka, Illinois, schools arguing in the *Journal of the National Education Association* for January, 1937, "The Case for Subjects in the Curriculum." Washburne, one of the earliest of the progressive school, very significantly concludes that the tool subjects (arithmetic, spelling, grammar, etc.) are handled more effectively in separate classes.

That the essential features of the activity program may be obtained *within* the separate course in arithmetic has been very nicely demonstrated by Henry Harap of Peabody College. He has organized arithmetic in the elementary grades around a series of practical activities—the making of ink or planning a taffy pull party, to cite but two examples. He has shown that classes so taught maintain a high position on standard tests on the fundamental processes despite the abandonment of formal textbooks and practice materials. Dr. Harap's main theses seem to be that *the pupil should learn each step in arithmetic as it arises in life-like activities* and that *learning goes on most effectively when the child enjoys what he is doing.*

The discussion of this question may well close with citing the conclusions reached by Paul Hanna, and others, in Chapter V of the *Tenth Yearbook of the National Council of Teachers of Mathematics*, entitled "Opportunities for the Use of Arithmetic in an Activity Program." The substance of these conclusions is that:

> Children should be taught arithmetic through meaningful and purposeful activities but that there should be, in addition to this, direct instruction in the skills of arithmetic in a separate period.
>
> The activity program provides insufficient experience for fixing skills but activities are useful in providing the experience that gives meanings to number relations.

2. *In what grade should formal arithmetic begin?*

Opinion is almost evenly divided on the question just proposed. One group, impressed by the findings that children have a considerable number of arithmetical concepts at school entrance, argues for immediate building upon this foundation by a carefully planned program in grades one and two. A second group, although not denying these experimental findings, holds that the postponement of formal work until the third grade provides additional mental maturity and that the pupils beginning at this later period achieve results fully equal to those beginning in early grades.

Two points must be borne in mind in evaluating the evidence which seems at first sight, quite contradictory. First, we must bear in mind that the issue is centered on the beginning of *formal* work in the fundamental operations. The question of whether *experiential* backgrounds for number concepts should be provided prior to the third grade is left open. Secondly, neither school of thought denies the experimental work of the other group. The question is purely one of the significance of the findings. Hildreth summarizes the situation well on pages 160 and 161 of her book entitled *Learning the Three R's*, when she writes:

> Children bring with them at school entrance a considerable store of number knowledge. Many achieve computation skill incidentally.

One child in six learned all the number combinations through nine plus nine in three months as the result of playing dominoes every day. Before the age of school entrance, children have long been accumulating on their own initiative a considerable store of number facts. *This fact would suggest that the child may safely be trusted to go on for some time with the added impetus the school affords, collecting number information from his experiences. Distaste for arithmetic, along with a narrowing of the child's opportunities for acquiring number sense in a natural way, may result from a heavy drill program in arithmetic in primary years.* (Underscoring mine.)

Detailed evidence of the very considerable knowledge of number facts and concepts was published by Buckingham and McLatchy in the *Twenty-Ninth Yearbook of the National Society for the Study of Education*, 1930, pp. 473–524. These two authors, and several others, have collaborated in a symposium under the direction of Brownell in the journal, *Childhood Education*, May, 1935. In both of these publications Buckingham takes a stand directly opposed to that of Hildreth, just cited. After reviewing the evidence that entering pupils have a rich fund of knowledge of counting , number facts, and even simple fractional relationships, he states:

We should begin the teaching of arithmetic as soon as he (the pupil) *comes to school.* (*Childhood Education*, May, 1935, p. 343; italics his.)

We must bear in mind the concept that Buckingham has of the kind of arithmetic he is advocating for grades two and three. This is best accomplished by another quotation from him in the section entitled, "*What Kind of Arithmetic Do We Mean?*"

Among the many divisions which we may make of arithmetic we may recognize concrete and abstract areas. Counting objects, reproducing, matching, and identifying numbers by means of objects, solving verbal problems by actual measurement with a ruler or with pint and quart cups, reaching conclusions on the basis of one-to-one correspondence, dramatizing a quantitative situation with a consequent decision as to its meaning, putting together or adding groups of objects, taking them apart or subtracting, playing games which require the use of number ideas, manipulating objects or pictures or number patterns for a quantitative purpose—all these are as truly "arithmetic" as abstract number facts and processes (loc. cit., p. 340).

Buckingham has also pointed out that the postponement of arithmetic represents a tendency opposed in direction to the downward shifting of every other school subject, as is illustrated by what has happened in the cases of geography, history, and the natural sciences.

It is clear that the question, "In What Grade Should Formal Arithmetic Begin?" dodges the real issue. If we focus on the word "formal," no one will object to the postponement until grade two or three. But, if we center our attention on the idea of "What Kind" of arithmetic in grade one, the great majority of educators will go along with a first grade program that builds upon the concepts that the child brings with him to school and gradually extends and enriches the several types of concrete activities which Buckingham has so well enumerated in the quotation given above. *So far as the first grade is concerned we may safely say: "formal arithmetic, no!; concrete number experiences, yes!"*

3. *What changes in grade placement are necessary?*

There is no question that a very considerable overhauling of the grade placement in arithmetic is under way. In fact, the present decade has already produced marked shifting to higher grades. More shifting will take place in the decade to come.

The Committee of Seven, beginning with its preliminary report in the *Twenty-Ninth Yearbook of the National Society for the Study of Education*, 1930, pp. 641–670, has published almost a score of overlapping and more or less repetitive accounts of its findings. These investigations, admittedly inconsistent at times, taken in their entirety suggest an average upward shifting of about one school grade for each of the major topics in arithmetic. It is gratifying to note that courses of study and textbooks have gradually been revis-

ing their grade placements in an upward direction, although it is to be noted that Brownell, in particular, has attacked these experimental findings on the score of defects in the experimental procedures.

There is a logical issue that disturbs the careful thinker on the subject of grade placement in mathematics. I refer to the possibility that the apparent misplacement of many topics may quite as logically be ascribed to faulty methods of instruction as to the too-early introduction of a given operation.

Let us take long division as an example. There are at least three reasons for choosing this topic. First, it is one of the admittedly "hard spots" in arithmetic. Secondly, several carefully controlled experiments have attempted to improve the methodology for presenting this topic. (I refer here to the work of Findley, Beall, Grossnickle, as well as that of the Committee of Seven.) Lastly, the present practice of abandoning the distinction between "short" and "long" division results in a simplification of the teaching to a single algorism, with resulting economy and the elimination of interferences in learning.

Two of the four studies (Findley and Beall) suggest that it is possible to retain long division in high fourth grade, one (Grossnickle) would locate it in low fourth grade, and only one (The Committee of Seven) suggests fifth grade or higher.

I am not so much concerned with the issue of fourth grade vs. fifth grade for long division, as such. Indeed, I will concede freely that all or most of so-called "long" division should be introduced in grade five. *The disturbing element in the situation is that there is a real danger that we shall adopt, wholesale, the simple and often thoughtless expedient of solving all our curricular problems by an upward shifting of mathematical topics.* If this should happen to the extent that emphasis on experimental work toward improving instruction should be relaxed, education will lose much more than it will gain by these changes. It is probably safe to reason from the analogy of the wide range of individual differences in pupils to the existence of a similar range of differences in teaching ability among teachers. Thus, a poorly prepared teacher and a slow class may mean failure of mastery of a topic in grade five while a good teacher and an average class will succeed brilliantly with that topic in grade four, or even lower.

We may leave the matter of grade placement in arithmetic by pointing out two facts to be borne in mind: First, that present tendencies in shifts in grade placement are running counter-current to the downward shift in the other elementary school subjects; secondly, that upward shifting should be the last resort, not the prime move in adjusting the curriculum to the child.

Mathematics in the Next Decade

It is trite to remark that prediction is hazardous. In this instance, however, certain present tendencies will almost certainly continue. The progressive education movement, so far as it concerns mathematics, will undoubtedly continue to accumulate facts which suggest activity units for the integration of mathematical processes around social rather than mathematical topics. This will be particularly true in the junior high school where the point of view will be that of the mathematics of the *consumer* in ever increasing degrees. Instead of such topical sequences as rectangles, angles, circles, the Rule of Pythagoras, square root, etc., we shall find an organization based upon such topics as ways of earning a living, budgeting the family's income, buying on the installment plan, planning a savings program, etc. Within these general spheres or centers of home, community, and national activity will be found units or activities of genuine social significance.

The issue of grade placement of topics will continue to occupy our attention although this issue may become somewhat less acute in the next decade with the greater realization that shifting of topics

is often merely an expedient. It is possible that a counter-movement may set in with a return to an emphasis on the improvement of instruction and a socialization of the content of mathematical programs.

It is my opinion that the next ten years will witness a critical struggle between the school of thought devoted to the drill theory of instruction and the school of thought that would provide varied, concrete and extensive background experiences with numbers prior to formal experience with abstract numbers. Meanings rather than drill will be the keynote of future discussions and the objective of instruction in mathematics. The practices in teaching will undoubtedly reflect the present controversy in academic psychology between the trial-and-error theory of learning and the learning-by-wholes theory which provides a much larger place for insight and meanings in learning than a psychology based upon animal experimentation suggests. It seems probable that "Meanings" will be the peg on which the major discussions of mathematics in the "forties" will hang.

If we have failed in our teaching of arithmetic, it is because we have failed to view the opportunities that mathematics offers for a wider realization of life. It is because we attempted to fit the child into an unreal world of abstract numbers, lines without end, surfaces without thickness, and symbols without inherent meanings. It is because we have not distinguished between the hieroglyphics of arithmetic notation and the experiences to be derived from contacts of eye, hand, and other sense organs, with the living objects and inanimate materials of a normal environment. So long as children can add one-half and one-half on paper but cannot fill an ordinary drinking glass half full of water with approximate accuracy or cut a pie into quarters with the degree of skill needed at the meal table, we must plead guilty to the charge of failure to raise mathematical instruction to the functional level.

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MEMBERS whose subscriptions expire with the May number have received notices of the fact together with a request to renew their subscriptions at once. These notices were sent out early to forestall the competition of summer and vacation needs which sometimes cause renewals for THE MATHEMATICS TEACHER to be sidetracked for several months.

It is a great service to the business manager to know *now* what funds he has to work with and how many subscribers he can count on for the coming year. Much of the work of editing and producing a professional journal has to be arranged for months in advance of the actual issues. It is extremely difficult, if not impossible, to make these necessary arrangements unless the support of our members is assured. It is for that reason that members are asked to send in their renewals and $2.00 fees promptly.

Since there are no issues of THE MATHEMATICS TEACHER in June, July, August, and September, subscriptions which started with the October 1938 number close with the May 1939 number. Those who became members last fall, starting with the October number, are also urgently asked to make their renewals now for the coming year in the light of the considerations outlined above.

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Contributions of Commerce to Mathematics

By J. A. G. Shirk
Kansas State Teachers College, Pittsburg, Kansas

Sometimes the workers in one field of learning are unaware of the influence of other fields upon their own, and they erroneously believe that all the important contributions to its advancement have been made by individuals whose principal interest has been along that particular line. Probably this conception is due to the great progress that has been made during the nineteenth century in almost all phases of human activity, thus making it difficult for any one to make further contributions except those who have delved deeply into that special realm.

While today most fields have already been extensively developed, there are still a few lines of thought in which relatively little has been accomplished. The frontiers of these new realms are rather close to us, and very important contributions are not infrequently made by workers with an established reputation in some older field.

Mathematics, as one of the oldest sciences, has long since pushed its frontiers so far away that investigators in one division of mathematics are unable to follow intelligently the researches in other phases. Henri Poincaré is often mentioned as the last generalist in mathematics. At the time of his death in 1912, no living mathematician was able to wear his mantle of universal mathematical knowledge and attainments. As we scrutinize the research activities of the prominent mathematicians of today, we find it relatively easy to list each one as being rather definitely confined to a limited number of mathematical domains. The aspiring devotee of mathematics is able to acquire only a broad survey of the several phases of the subject, after doing which he must confine his efforts to some particular type of research in order to be able to add to the vast store of mathematical knowledge.

It is the purpose of this article to set forth the influence that commerce has had upon the development of mathematical ideas, and also how such ideas have been disseminated through the operations of commercial transactions.

The beginnings of mathematics are shrouded in the veil of the prehistoric unknown, and we can only surmise how the elementary mathematical notions were evolved. There are still a few very backward tribes whose manner of thought is probably not much different from the uncivilized ancestors of those early civilizations whose mathematical attainments we now know quite well. Primitive man early learned some notions of geometrical form and design in the making of pottery with the designs painted upon those objects. Counting probably came at a still earlier stage in the enumeration of objects or animals possessed by the tribe or by individuals.

A knowledge of the number of fighting men of the tribe as compared to the number of their enemies was most essential to the continued existence of the tribe. The exchange of various articles in the beginnings of commerce made a knowledge of the size, shape, and number of such articles quite desirable. The simple operations of addition and subtraction must necessarily have been developed in connection with these commercial transactions. The operations were performed by the use of some type of counting device. In very simple cases where the numbers involved were small, this counting was probably done by the aid of the fingers and toes. The operations with larger numbers made some other devices necessary, and the sand board, wax tablet, and abacus were developed to provide concrete means of performing arithmetical operations.

The earliest civilizations employing mathematics were the Egyptian and Bab-

ylonian. The Egyptian mathematics was not developed through the impetus of trade, but rather as an aid to the construction of pyramids, obelisks, and temples, and in connection with the re-surveying of land after the sediment from the overflow of the Nile river had covered up boundary marks or when the river had washed away portions of fields. There was also an extensive irrigation system in which the construction of canals and ditches made elementary leveling processes imperative.

Egypt carried on very little commerce with the rest of the world previous to its conquest by foreign kings about 1600 B.C. The country had sufficient resources to meet all its needs, hence trade with other nations was not necessary. Also the Arabian desert had to be crossed to make contacts with the other great civilizations of that period. At a later period, the mathematics of Egypt was given to other peoples by means of traders who came to Egypt for the purpose of exchanging their wares for the products of the Nile Valley.

Babylonia on the other hand was an aggressive commercial nation. Situated midway between the nations of southern and western Asia, it was in a position of advantage to carry on the trade of the world. Great mercantile firms were developed, and almost all of the principal business procedures now in use were developed by this resourceful people. Contracts, mortgages, notes, partnerships,—all expressed with capital of dates, date wine, flour, oil, barley, and sometimes silver—were recorded on tablets of soft clay. These tablets were then hardened by baking which made them very durable. Records of the commercial transactions of some of the large firms over a period of several generations are found in the mounds which mark the ruins of once populous cities.

Interest rates were probably standardized by law or custom, as almost no mention is made of the rate of interest charged, although there is abundant evidence of the loaning of capital and the payment of interest for loans.

The laws of Hammurabi, codified about 2100 B.C., show very clearly the commercial development of Babylonia. These laws contain many provisions concerning the relations of agents to employers, losses of goods due to hazards of travel, penalties for fraud or violations of agreements, and many other business situations.

The commercial mathematics of the Babylonians was much more extended than is commonly thought. Recent investigations have brought to light ingenious operations with fractions and even the solution of some problems that are fundamentally algebraic in character. Possibly the Hindus obtained some of their first concepts from the Babylonians. The place-value idea seems to have originated with the Babylonians and next appears in Hindu mathematics. Unless this idea was independently developed by the Hindus, the commercial contacts of the Hindus with nations whose culture was influenced by Babylonian concepts must explain the reappearance of this very important concept in number representation.

The greatest commercial nation of the ancient world has left no records that compare in completeness with those of Babylonia. From 1600–1000 B.C., most of the trade of the world was carried on by the Phoenicians. Caravan routes led eastward even as far as India, and ships went regularly to Spain and to England for silver and tin. Undoubtedly most of their trading was merely barter, but they must have used considerable arithmetic in such extensive commercial enterprises. They were the greatest ship builders of antiquity, not only making them for their own use, but selling them to other nations. Solomon bought ships from the Phoenicians for use in his trade with Arabia. These ships were transported in separate pieces to the Red Sea, where they were assembled. The marking of each part must necessarily have involved ideas of position and number that are essentially mathematical.

The Greeks were early driven into commerce by the lack of certain products in

their home land. In reaching out to other lands, they established cities in Asia Minor, in Italy, and on many islands of the Mediterranean. Miletus in Asia Minor was the first to become important. Thales was one of its most prosperous merchants. He made many trips to Egypt for commercial purposes, and perhaps went to Babylonia. He became interested in the geometry of the Egyptians and the astronomy of the Babylonians.

The geometry of the Egyptian was partly intuitive and partly empirical. There was no thought of demonstrating formulas but only of evolving processes of finding the lengths and areas necessary to their peculiar agricultural conditions and for the building operations which they conducted so extensively. Thales was the first man to appreciate the desirability of demonstrating geometrical theorems, and while he can be credited with proving only five or six theorems, he deserves great honor for laying the foundations for demonstrative geometry. To commercial activities belongs the credit for the transformation from the empirical geometry of the Egyptians to the demonstrative geometry of the Greeks, since it was through commerce that Greek merchants first came into contact with the Egyptian mathematics.

We have almost no records of Greek arithmetic used in connection with their extensive commerce. This is due to the fact that the Greeks put utilitarian mathematics in an entirely different category from theoretical work. In their estimation, the only mathematics worthy of preservation was that type which was obtained as the result of logical thought.

It is almost universally true that commerce and its attendant wealth produces the conditions under which the arts and sciences can flourish. Greek centers of art and learning followed the centers of trade from Greece to the coast of Asia Minor, to the island cities of the Mediterranean, to the Greek cities in Italy, and to Alexandria. Thus did trade and commerce stimulate the development of logical mathematics among the Greeks and cause its spread over almost all of the civilized world of that era.

The Romans were at first predominantly an agricultural people, but their trade led them into an extensive commerce, and they acquired a reputation for honesty and fair dealing that made them soon the commercial leaders of the world. Roman ships went to Africa, Spain, France, India, and even to China. Practical arithmetic and constructive geometry were the only types of mathematics that appealed to the Romans. The risks encountered by storms and pirates led to the insuring of ships and cargoes against these hazards. This seems to be the beginning of maritime insurance. Again commercial operations had produced a new aspect of arithmetic. After the fall of the Roman empire, the practice of insuring ships and cargoes was discontinued until the rise of the Italian cities of Venice, Pisa, Genoa, and Florence.

After the fall of Rome, Europe passed into a dormant intellectual condition generally called the Dark Ages. Italian merchants from the cities mentioned went to Syria, Northern Africa, Flanders, London, and many other remote regions. The Arabians had settled down to develop the countries captured by them. They absorbed the culture of the captured peoples and became intensely interested in art and science, particularly mathematics. The Italian merchants came into contact with these Arabian peoples and learned of the ancient mathematics that had been treasured by the Arabian scholars.

Through trading operations, the Arabians had come into contact with the Hindus of India, and received from them the Hindu system of numerals with that scheme of representing numbers. The Arabians also obtained algebra from the Hindus who had solved both the linear and the quadratic equation. Arabian mathematics was introduced into Italy through the outstanding work of Fibonacci, whose father was agent for Pisa in its trade with the Arabian city of Bougia in

Northern Africa. Fibonacci introduced the new numerals into Europe, along with the algebra which the Arabians had received from the Hindus and had developed quite extensively. Again the spread of mathematics is due directly to the operations of commerce. Since Fibonacci introduced the new numerals into Europe from Arabian civilization, they were known as the "Arabic Numerals," and not until recent times have they been given the name of Hindu-Arabic Numerals in recognition of the part that one people played in originating these numerals and the other people in transmitting them to Europe. This important transmission was made about 1200 A.D., and from that date most of the important advances in mathematics were made in Europe.

The great fairs of medieval Europe led to the development of many new problems in arithmetic. Problems of exchange, customs duties, profit and loss, drafts, and deferred payments became of great importance. Arithmetics were written to expound the methods of solving these problems. Among the most important of these arithmetics was Borghis' arithmetic published at Venice in 1484, Widman's arithmetic in Germany in 1489, Adam Riese's arithmetic, published also in Germany in 1522, and the English arithmetic of Robert Recorde, published about 1540.

In these distinctly commercial arithmetics are found the first use of many mathematical symbols. Widman introduced the plus and minus signs used today. These were introduced into England by Robert Recorde, who was the originator of the equality sign.

During this period the risks of trade led to the revival of maritime insurance by a group of London merchants who met in Lloyds' coffeehouse in London. This was the origin of the great insurance business of Lloyds of London. Fire insurance rapidly followed, and other forms of insurance were later developed until now almost anything can be insured against any type of loss.

Perhaps the two most outstanding contributions of commerce to mathematics were the interpretation of negative numbers as losses by Fibonacci and the invention of decimals by Stevin. Before this useful interpretation of negative numbers was made by the merchant of Pisa, negative numbers were not considered as having any practical value. When negative answers were encountered in solving any problems, these answers were rejected as being of no practical significance. Their very name indicate this attitude toward them, being derived from the Latin word "negare" which means to deny. Since that time so many other useful interpretations of negative numbers have been made in science and engineering that they are now thought of as being just as important as the positive numbers.

Before the invention of the Hindu-Arabic numerals, fractions were expressed in a variety of ways to simplify somewhat the operations with them. The need for simpler computations in commerce was the impelling motive in these ancient schemes of representing fractions and working with them. The Egyptians used what are known as unit fractions. In these fractions the numerators are always unity and the denominators are positive integers. They found it convenient to have one fraction not of this type which was 2/3. By means of these unit fractions they could express any fraction as an example, $6/7=2/3+1/7+1/21$. The use of the special fraction 2/3 made it possible to represent many fractions in a simpler manner than by unit fractions alone since without its use $6/7=1/2+1/6+1/7+1/21$. The Babylonians used fractions whose denominators were multiples of 60, in a manner somewhat suggestive of fractions whose denominators are powers of 10, which in reality are decimal fractions with the denominators omitted and with the numerators arranged in definite positions to indicate their respective denominators.

The Hindu-Arabic numerals were introduced into Italy by Fibonacci near 1200

A.D., and it was almost four centuries later before Simon Stevin completed that number system by the invention of decimal fractions. Until this was accomplished there was not so great an advantage in using the Hindu numerals, but after decimals were invented and a good symbolism was developed, it was only a short time until the use of other systems of numbers was entirely discarded in commerce. Stevin lived in the Netherlands and was inspector of the dykes, quartermaster general of the army, and minister of finance. These positions are evidence of his executive ability in financial and commercial affairs. It is evident that Stevin arrived at the discovery of decimals by combining the essential features of sexagesimal fractions with the place-value idea of the Hindu-Arabic numerals. After trial by many prominent practical men, the decimal system was recommended by them as much superior to all short cuts or devices then in use for using common fractions. Stevin showed how all the computations met in business could be performed by integers alone without the aid of fractions. He suggested that most practical work could be made much less complicated if the units of measure would be divided into tenths, hundredths, etc. This was done in the formation of the Metric System of weights and measures, and is now being practically used in the English System. The surveyor now subdivides the foot into tenths and hundredths and the machinist subdivides the inch in a similar manner. The sizes of electrical conductors are decimally expressed in mils. Further evidence of the value of the decimal system is the manufacture of surveying instruments on which the degrees are divided into tenths and read by verniers to hundredths. By the decimalization of units used in special vocations, it will be possible to utilize the great conveniences of the decimal system before the Metric System is universally used. Four centuries elapsed between the introduction of the Hindu-Arabic numerals into Italy by Fibonacci, and it now seems as if another four centuries may elapse before the final adoption of a complete decimal system in all weights and measures. When that is accomplished the vision of Simon Stevin will have been realized.

Another most important contribution of commerce to mathematics was in the invention of calculating devices. Some types of calculating devices were developed especially for architects and engineers but others were devised as aids in making the computations encountered in business. Those devices particularly adapted to commercial activities are generally known as calculating or computing devices and machines.

If relatively simple in construction, they are called devices, but when their construction and operation becomes more complicated, they are called machines. Among the calculating devices extensively used were the wax tablets and sand boards of the early Greeks, and the various forms of the abacus used by the Greeks, Romans, medieval Europeans, Chinese, and Japanese. Loose counters laid on lines laid off on a board were used to represent numbers on the medieval counting board, which is really a form of the abacus. The operations on the abacus quite generally gave place to operations on the counting board, and this in turn was used less as operations with the Hindu-Arabic numerals became more generally known. The greatest disadvantage in connection with the use of either the abacus or the counting board was that some numbers used in the computation disappear as the work proceeds, hence making the reviewing or checking of the work very difficult or even impossible.

The invention of the adding machine by Blaise Pascal in 1642 as an aid to his father in auditing the government accounts at Rouen, France, was probably the most important step in the development of calculating machines. The same principle is used in most of the multiplication machines later developed almost si-

multaneously in England and in Germany. Many special forms of computing machines have been developed for use in banks, mercantile establishments, and manufacturing enterprises.

The influence of commerce on mathematics may be briefly summarized under the following important accomplishments:

1. Development of prehistoric number concept.
2. Development of elementary mathematical operations.
3. Dissemination of mathematical knowledge by early traders.
4. First interpretation of negative numbers that gave them practical significance.
5. Invention of decimals.
6. Invention of calculating devices.

When viewed as a whole, the influence of commerce on mathematics is seen to have been most stimulating. Without the urge for better and more rapid ways of making the calculations of commerce, the progress of mathematics, to say the least, would have been very much retarded.

Primary Facts of the History of Mathematics

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According to a recent article on "Curricular offerings of State Teachers Colleges" published in *School and Society,* volume 48, pages 251–52, twenty-four of the fifty-three colleges located in twenty-eight states and selected at random, were then offering courses on the history of mathematics. It may be assumed that a number of the teachers of this subject are keenly interested in what may be reasonably regarded as the primary facts of this history in view of the very large number of the facts relating thereto which are available in periodicals and in textbooks. While it cannot be reasonably assumed that the views on this point are uniform, it may be helpful to some to state here a few opinions and some reasons in support thereof by one who has had a long experience.

It is commonly acknowledged that mathematics is largely an abstract subject from its elements upwards. The questions naturally arise when it became abstract and whether there has been a decided change as regards its relative abstractness during the period covered by the extant literature relating thereto. As a clear illustration of abstract numbers, we may cite the numbers 1, 2, 3, etc., and as a clear illustration of concrete numbers, we may mention the numbers 2 feet, 3 men, etc. An important publication relating to pre-Grecian mathematics appeared several years ago under the title *Vorlesungen über Geschichte der Antiken Mathematischen Wissenschaften* by O. Neugebauer (1934). On page 203 and elsewhere, it is noted therein that one of the chief objectives of the ancient Babylonian mathematics, which is the outstanding pre-Grecian mathematics, was the use of the fundamental operations in the number aggregate composed of the abstract positive rational numbers.

The extant mathematical literature contains many evidences of a growing and early tendency towards generalizations but with respect to abstractions, it is not so easy to see that there has been a decided relative change. While greater and greater abstractions appear, there is also a decided advance in concrete or applied mathematics. It has been known for many years that the first table in the Egyptian *Rhind Mathematical Papyrus* (about 1700 B.C.) relates entirely to abstract numbers, but that these numbers were not explicitly defined until recent times. Pure and applied mathematics supplemented each other during the entire period covered by the extant mathematical literature and no clear line of demarcation between the literatures of these subjects exists. It is, however, clear that the common multiplication tables belong to pure mathematics while tables of weights and measures belong to applied mathematics. Both of these types of tables are very ancient.

The main historical fact in what precedes is that abstract mathematics is as ancient as the earliest extant mathematical literature and that it would be difficult to prove that mathematics has become relatively more abstract since that time because the central feature of the oldest Babylonian mathematics is the treatment of abstract numbers, and this mathematics included the finding of a root of a quadratic equation by methods which are still used for this purpose, including completing the square. A primary fact of the history of mathematics is that special symbols for certain fractions, such as 1/2, 2/3, 3/4, etc., appear in the most ancient mathematical literature and that there is now a common name for the first of these fractions which is not connected with the name for 2. Hence it cannot be proved historically that the integers are older than some of the common fractions although this has often been assumed to be a fact.

Mathematics is, like truth, intrinsically universal, and it is unbounded, undivided, and undefined, but there is a considerable amount of human knowledge which is now universally regarded as mathematics, and the names arithmetic, algebra, analysis, and geometry suggest somewhat vaguely, special types of developments, but there is no essential difference between the first two of these subjects, such as is stated under the entry "algebra" in the second edition of Webster's *New International Dictionary* (1938). One of the primary facts of the history of mathematics is that such misstatements abound in literature and their implications frequently raise questions of wide bearings. Hence the effort to correct historical statements frequently constitutes a very fruitful exercise in a course in the history of mathematics. Truthful statements will only stand out the more clearly as a result of such efforts and critical examinations vitalize history.

The Greek contributions to the development of mathematics are especially interesting to us since they are the source of many of the later mathematical developments in Western Europe, and these, in turn, are the source of much of the American mathematics. Unfortunately, the Greeks gave comparatively few references to the earlier developments. In particular, Euclid's *Elements*, which occupy a central position in the history of Greek mathematics, contain no historical references, and hence it is now difficult to trace the developments found therein to their sources. During the past decade, it has been found that the ancient Sumerians and the ancient Babylonians deserve more credit than was given to them by the early Greek writers. No other large subject is built up rigorously from so few fundamental principles as mathematics, and this fact was first largely exhibited by Greek writers when they introduced systems of postulates and discussed the number concept as such.

A primary fact of the history of mathematics is that it is a rapidly growing subject and, hence, the teacher of this subject should be especially careful in choosing his works of reference. For instance, in 1898, the very widely and favorably known *Encyklopädie der mathematischen Wissenschaften* began to appear. The opening number is devoted to Arithmetic and contains a considerable number of historical notes. On page 12 there appears a note on negative numbers which involves a number of statements which are now known to be incorrect. For instance, it is here stated that the calculating with negative numbers began with R. Descartes, while in the French edition of the same work, which appeared in 1904, it is correctly stated on page 35 that the actual calculation with negative numbers came after R. Descartes. The same German note also states that R. Descartes used the same letter to represent sometimes a positive and sometimes a negative number, while it is now known that J. Hudde first did this in 1657.

It is a somewhat singular fact that the historical advances relating to the most ancient mathematical developments have been the most rapid in recent years. This has been largely due to the deciphering of cuneiform texts relating to the ancient Babylonian and Sumerian mathematics. Comparatively little has been done with a view towards selecting from these and other advances those facts which are of primary importance because they enable the student to trace their implications and thus to secure a large number of additional facts from a comparatively small number of given ones. The study of such implications tends to vitalize the whole subject. Not very much can be gained from accepting the conclusions of others except in so far as these conclusions serve as a basis for other conclusions on the part of the student. Statements relating to the history of mathematics are frequently rich in their implications and hence they may become fertile ferments in the minds of students. The acquisition of such ferments is highly desirable.

A primary fact in the history of mathematics is that mathematical developments are not homogeneous in the sense that closely related facts were not always developed about the same time, or that the

mathematical advances always become common property soon after their discovery. Mathematicians usually worked in isolation and probably comparatively few of their discoveries were made known to others and a small part of the latter attracted wide attention at the time when they were made. Even the use of the fundamental operations of arithmetic seems to have been extremely slow and no individual can now properly be given credit for its early development. It represents a national achievement rather than the achievement of a few individuals. In other cases, such as the solution of the quadratic equation, it is possible to trace steps in advance extending through about four thousand years of time and through many different countries. None of the American aborigines seem to have solved quadratic equations before they came in contact with the European settlers, and this is an index of the backwardness of their civilization compared with that of some other early races. The early settlers brought with them higher attainments than they found.

It would be a mistake to assume that the mathematical knowledge once acquired by certain members of the human race always remained a permanent possession thereof. On the contrary, many mathematical facts were developed a number of times before they became permanent property and the history of these rediscoveries is usually difficult since one cannot be certain in many cases that the later work was uninfluenced by that which preceded. For instance, the use of negative numbers by the Hindus may possibly have been influenced by the earlier use of these numbers by the Babylonians, and the former use may possibly have had some influence on the later introduction of these numbers into Europe. It seems clear, however, that these numbers were not fully understood before about the beginning of the nineteenth century when Caspar Wessel (1745–1818) and others laid a solid foundation for their treatment as well as for the treatment of complex numbers.

Much of the elementary mathematics may be regarded as a kind of thought currency which is used for convenience by many who do not stop to consider its real nature just as in financial transactions people learn to use the currency of a country without always considering the basic principles which give value to it. A very important source of progress in mathematics is the fact that the rules developed for the solution of special cases have often been found later to be much more general than they at first appeared and that concrete numbers can often be replaced by abstract ones. In fact, the use of abstract mathematics has been simplified by the frequent convenience of speaking abstractly when one thinks concretely. It is thus that the solution of equations which were at first constructed on the assumption that they have real positive roots only gave rise to a great extension of the number concept. The complex numbers came into mathematics without being invited or appreciated at first, but they brought thereto one of its richest treasures and some of its most fruitful seeds for further development. Their introduction was entirely due to the mathematicians of Western Europe.

One of the primary facts of the history of mathematics is the discovery of its most fundamental concepts and the study of the developments based thereon. The concepts of number and geometric figure are doubtless basic, and the concepts of equation and limit presented themselves early in the development of our subject. The concepts of function and group took clear form much later but have since then manifested their widespread influences. These six concepts may be made centers of historical interest in order to coordinate a large number of the special developments with which the history of our subjects abounds. Such a coordination simplifies the study of the history of the diversified and extensive developments which confront those who aim to secure an intellectual penetration into the rich heritage which mathematics involves and which, as teachers, we are privileged to transmit.

An Experimental Study of the Relation of Homework to Achievement in Arithmetic

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The uncertain value of homework is a topic which has been debated for years and the attention it has attracted recently reflects the differences in underlying attitudes. The traditional view regards homework as a valuable and perhaps indispensable supplement to classroom instruction. Many are of the opinion, however, that its worth has been exaggerated and that the undesirable consequences far outweigh any benefit that compulsory homework may entail.

Most of the appraisals of homework have been based on the opinions of parents, teachers, and pupils. The reactions of these groups have varied and marked inconsistencies are noteworthy. These are not surprising in view of the many conditioning factors involved such as the kind and amount of homework, the age and abilities of the pupils, the facilities available, and many others. The influence which these conditions exert precludes the possibility of settling the question by an unqualified answer, or through any single approach to the problem as a whole. One of the significant facts revealed by several studies is that teachers are more opposed to homework than are parents or children.

Experimental studies of homework have been less numerous than would be expected from the attention given the problem in other ways. The most extensive of the quantitative investigations are those of Steiner and Di Napoli. Steiner[1] noted greater improvement in both phases of arithmetic in the group given home study assignments despite the fact that, as Steiner remarks, the homework was not as correlated with classroom practice as it should have been. The more recent study by Di Napoli[2] employed groups of fifth and of seventh grade pupils. Compulsory homework in the fifth grade enhanced achievement to a greater extent than prevailed in the seventh grade, but none of the differences was very large. Di Napoli's results led him to conclude that achievement was not benefited to the extent that the proponents of homework have claimed, and he urged that the amount of compulsory homework in the elementary schools be curtailed. His data pertain to but one phase of the question since he did not take into account other possible advantages such as those arising from the development of habits of independent study.

The present experiment was conducted in seven parochial schools during the school year 1935–36. All of the classes had regularly assigned homework during one term and no homework during the other. The classes exchanged roles at the end of the first semester. This rotation procedure was employed since it furnished better control over several factors than would have been possible through other procedures.

At the beginning of the year all pupils were given the Otis Group Intelligence Test and the New Stanford Achievement Test. Two other forms of the New Stanford were administered at mid-year and

[1] Steiner, M. A., "Value of Home-Study Assignments," *School and Society*, XL, July 7, 1934, 20–24.

[2] Di Napoli, P. J., "Homework in the New York City Elementary Schools," Teachers College, Columbia University, *Contributions to Education*, No. 719, 1937, 60.

in June. Informal tests in arithmetic were conducted from time to time but the results of these have not been incorporated into this report. The data considered are the scores on the arithmetic computation and problem exercises of the New Stanford.

At the beginning of the fall term conferences were held with the teachers who were fully informed regarding the purpose of the study. No homework was to be assigned in any subject except arithmetic and then only during one term. Emphasis was placed on the necessity of making the assignments definite and of providing for individual differences. The greater part of the homework was connected with the newly introduced topics in the seventh grade, but some dealt with remedial work carried on simultaneously with the other instruction. The tasks assigned were not to exceed one-half hour in duration but other details were left to the discretion of the teachers. No attempt was made to substitute an equivalent amount of practice for those enrolled in the control groups. Needless to say, pupils were held responsible for the work assigned and in other respects as well, the homework adhered to the conventional pattern though with more uniformity among these classes than could be expected among classes at large.

The rotation method dispensed with the necessity of equating the classes in all particulars since each class spent the same amount of time as a homework class and as a control class. However, the general intelligence of the two groups was approximately the same, and the initial arithmetic scores do not differ significantly when the averages are compared.

THE RELATION OF HOMEWORK TO ACHIEVEMENT IN ARITHMETIC

	GROUP A		GROUP B	
Number of classes	3		4	
Number of pupils	148		144	
	M	S.D.	M	S.D.
Intelligence test scores	105.55	9.20	107.71	9.17
New Stanford Arithmetic Problems				
September	79.35	10.65	81.88	10.57
January	85.27	12.56	86.35	11.22
Gain	5.92	(Control)	4.47	(Homework)
January	85.27	12.56	86.35	11.22
June	91.70	12.61	89.90	12.35
Gain	6.43	(Homework)	3.55	(Control)

Total gains: Homework groups: 4.47+6.43=10.90
Control groups: 5.92+3.55= 9.47
Difference = 1.43

	M	S.D.	M	S.D.
New Stanford Arithmetic Computation				
September	78.82	14.40	79.52	14.10
January	87.03	16.55	88.70	14.89
Gain	8.21	(Control)	9.18	(Homework)
January	87.03	16.55	88.70	14.89
June	92.85	15.22	93.45	12.43
Gain	5.82	(Homework)	4.75	(Control)

Total gains: Homework groups: 9.18+5.82=15.00
Control groups: 8.21+4.75=12.96
Difference = 2.04

The results obtained from the various tests are summarized in the accompanying table. The net gain attributable to homework is the difference between the gains of the homework and control groups. In problem solving the net gain is small, 1.43. The significance of this difference is overshadowed by the fact that one control group gained more than the corresponding homework group while the reverse held

true in the other comparison during the second semester.

The increases in computational skill are considerably greater than those that occurred in problem solving for the two groups. The difference between the control and homework groups favors the latter by 2.04 units of the Stanford test. Both homework groups gained more than the control groups but the differences are small.

Although Group A had slightly lower average intelligence and initial skill in arithmetic, they gained as much as the other group who had regular homework in arithmetic problem solving. No relation can be detected between the gains and the semesters or between the gains and average initial achievement in arithmetic. Examination of the gains made by the seven schools revealed equally large gains under both programs although the accumulated gain favors the groups having home assignments. Considering both phases of arithmetic, fourteen comparisons are made possible and the advantages divide rather evenly between the homework and no-homework groups. In eight of the fourteen comparisons, the greater gain is associated with homework. In the remaining six, the group that had no homework made more progress.

One of the unexpected results is the difference between the improvements in problem solving and computation. Although the homework consisted mainly if not exclusively in the solving of verbal problems, the greater test gains occurred in computation. This somewhat anomalous result suggests that whatever other differences exist must be attributed to conditions of learning other than those evaluated in this experiment.

The facts themselves provide no substantial foundation for the opinion that the homework gains are especially significant from the practical viewpoint. Perhaps, however, the accumulation of minute advantages would necessitate the adoption of homework as a factor contributing something to total accomplishment. An analysis of homework as such suggests that its contribution to achievement arises from whatever function practice serves. If the homework serves only this purpose its advantages are dubious.

The results are in harmony with those of Di Napoli and others[3] who have conducted similar inquiries. The interpretation of the evidence leaves some room for differences of opinion although the general trend confirms the widely held suspicion that homework is not an important factor in promoting achievement among seventh grade pupils in arithmetic.

[3] Vincent, H. D., "Experimental Test of the Value of Homework in Grades Five and Six," *National Elementary Principal*, XVI, 1937, 199–203.

THE originality of the human being depends both on heredity and on development. We know that individuality springs from these two sources. But not what part each of them plays in our formation. Is heredity more important than development, or *vice versa?* Watson and the behaviourists proclaim that education and environment are capable of giving human beings any desired form. Education would be everything, and heredity nothing. Geneticists believe, on the contrary, that heredity imposes itself on man like ancient fate, and that the salvation of the race lies, not in education, but in eugenics. Both schools forget that such a problem cannot be solved by arguments, but only by observations and experiments.—ALEXIS CARREL *in Man, The Unknown.*

A Mechanical Solution of the Cubic

By Robert C. Yates
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It is well known that the general cubic

(1) $$u^3+Au^2+Bu+C=0$$

can always be reduced to the form:

(2) $$v^3+av+b=0$$

by the transformation $u=v-A/3$. However, not so widely known[1] is the further reduction of this equation by the rational transformation $v=bx/a$ to:

(3) $$x^3-m(x+1)=0,$$

where

$$m=-a^3/b^2.$$

Thus any given cubic may be made to depend upon a *single constant* by means of a transformation which is rational in the original coefficients.

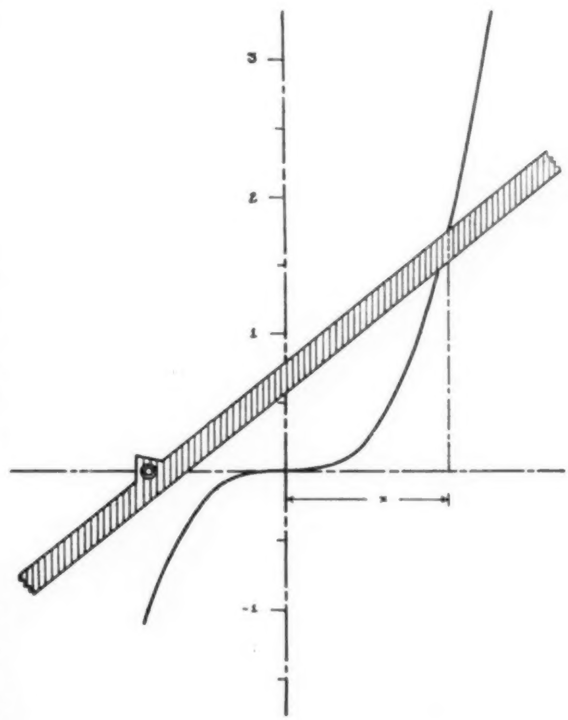
A point of particular interest follows directly from the graphical solution of equation (3). We replace this equation by the set:

(4) $$\begin{cases} y=x^3 \\ y=m(x+1), \end{cases}$$

whose simultaneous values of x also belong to (3). We have then, for *all* cubics, a fixed curve and a variable line. But this variable line has the distinct virtue of always passing through the point $(-1, 0)$. This is the feature that prompts the mechanical arrangement shown.

[1] But see: H. A. Nogrady, "A New Method for the Solution of Cubic Equations," *American Mathematical Monthly*, 44 (1937) pp. 36–38.

A cardboard strip is attached to the plane, on which the curve $y=x^3$ is drawn, so that its straightedge rotates about $(-1, 0)$. Its position, of course, is determined by the slope $m=-a^3/b^2$, measured directly upon the vertical scale. The root x is then determined by the perpendicular dropped from the intersection onto the horizontal axis. Needless to say, to a worker confronted with the task of solving a large number of cubics, this apparatus would prove of considerable value.



Have You Paid Your Dues?

The October issue of The Mathematics Teacher will be sent *only to those whose dues are paid*. The date of expiration of your subscription is stamped on the wrapper of your May issue.

The Role of Mathematics in the Twentieth Century Curriculum*

By WILLARD W. BEATTY
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I AM appearing here at the request of your president as a spokesman for some of the progressive group who have been much concerned with the mathematics program of our junior and senior high schools. It is only fair to confess that while I have taught some phases of high school mathematics myself, it is not as a pure mathematician but as a student of the secondary curriculum that I have approached the problem. For years I have found mathematics increasingly on the defensive.

While the public high school was considered a selective institution for preparing a few superior children for the further selective processes of a college education, the mathematics curriculum may well have served as an intellectual hurdle. Today when the general public has come to believe that a high school education is the right of every child, there is going to be no patience with any organization of the curriculum by which any one or two subjects are required of all and where failure in these subjects is presumed to be an adequate measure of general incompetence. One indication of the degree to which universal high school education has been accepted in this country is a report in a forthcoming issue of the *Ladies Home Journal* of a statistical survey of public sentiment which reveals that 91% of the women questioned take this objective for granted.

We shall not now take time to discuss the many reasons for the development of this public attitude. What is of equal or perhaps more importance is the growth of knowledge in significant other fields such as general psychology, social psychology, and sociology, which mathematical educators cannot ignore. Along with this has developed a general awareness of the fact that this generation, and many generations to come, face a rapidly changing economic world that calls for new patterns of quantitative thinking. The combination of these factors calls for a thorough re-examination of the part of mathematics teachers and of the role of mathematics in the twentieth century curriculum.

While sharing the impatience, therefore, of many laymen and school people who criticize the traditional courses in secondary mathematics, I strongly believe that mathematical thinking has a tremendous contribution to make to general culture. Let us then consider some of the attributes of mathematical thinking and of culture as we see it today.

By mathematical thinking I am referring to the quantitative relationships which exist through the vast realm of human relationships, as well as the more limited areas of the exact mathematical sciences represented by algebra, geometry, and the several disciplines which follow them. In fact, as mathematics has been taught in our upper schools, we have been peculiarly unsuccessful in conveying to our students of mathematics a capacity to recognize quantitative relationships and quantitative concepts when they are encountered outside of the classroom.

For the last fifteen years, therefore, I have been actively attempting to persuade the mathematicians of my acquaintance to take what I believe to be necessary steps to rectify this condition. Unfortunately, many mathematics teachers tend to take the position that mathematics is an all or none proposition. Mathematics must either be accepted in its traditional form or it disappears as a discipline, they say. That, I believe, is a

* An address given before the National Council of Teachers of Mathematics at Cleveland, Ohio, on February 24, 1939.

most shortsighted position, and from the standpoint of the place of mathematics in the high school curriculum, a most dangerous one. There will always be a small group of high school students who I hope will want and should have all of the experiences in abstract mathematics that it is possible to include in a high school course. When we single out these mathematical specialists and offer them a program of instruction commensurate with their capacity and suited to their needs, more and better mathematics will be taught in these courses.

But for the rank and file of the new masses entering our high schools, the traditional courses in mathematics have little significance. As a result, new material must be found, relating mathematical concepts to experience, or advanced mathematics will follow Greek out of the general curriculum. This would be unfortunate, for quantitative thinking can and should be made one of the student's most stirring experiences in arriving at a better understanding and eventual control over social and economic forces, as well as problems of the physical environment. Mathematics teachers themselves must have the vision and courage to approach the possible contribution of their subject for general education in new terms and patterns.

In speaking of mathematics in our culture, however, I am not thinking of the older view of the so-called cultural disciplines, but something broader and more dynamic. For example, the current year book of the John Dewey Society calls for a curriculum designed directly from the total culture. The first chapters of that volume define culture with an all-inclusive meaning:

> First, the external material civilization—the ways and means by which a people produce and distribute its physical goods, buys and sells, communicates, and the like—and in short, its total economic system.
>
> Second: beneath the obvious physical civilization the culture embraces the social institutions of the people—the characteristic family life, the government, the industry and business, the other economic and social organizations, the press, radio and other agencies of communication, the ritual of churches, lodges, schools, and colleges, the work of forums and other parliamentary procedures, the ritual of courtesy in social life, codified food habits, ways of dress, speech, recreation, and the like. The social institutions include also the language of the people, their ways of measuring, recording, and expressing facts, their use of science and art—all of these used as subtle instruments of thinking and feeling.
>
> Third: even more directive and formulating than the external economic civilization and the social institutions is the "psychology" of the people. The social arrangements of a people are created primarily by their drives, their attitudes, their ideas. What they have in their heads, what they most want, what they fear most, determine what they do and what they are. Their desires dominate their social psychology. To name only a few examples, there are the desires for personal security, for a better living, for social approval. But the social psychology of a people also includes the all-pervasive "climate of opinion" of the wider community, molded by such directive concepts and attitudes as freedom, equality of opportunity, justice, patriotism, and the like.
>
> There are, perhaps, other factors which play a determining part in making Americans what they are, but these illustrations of three phases—the external physical civilization, the social institutions, and the underlying psychology—explain sufficiently the sense in which we use the term culture.

This concept of the culture as the frame of reference for the curriculum is gaining wider and wider acceptance in educational thinking today, and this concept has definite implications for the teaching of mathematics.

For illustration, permit me to take you for a moment to the examination of a specific cultural-economic problem facing some of our Indian groups because we find there so many of our own white problems staring us in the face in sharper focus. I will cite the automobile because you can see the parallelism and make the transfer readily to our own curricular problems.

The time has come when many of our Indians are being exposed to the automobile. True, it is generally a third or fourth-hand, dilapidated machine, but the

Indian wants it and wants it badly. Like a child, he lacks an understanding of our money values. Like a child, he cannot wait and save up for the necessary cash. He wants the automobile today, just as our white brethren rush for the newest and latest model before their present car actually needs replacing. In his haste, he is likely to sell today for $40 his interest in a crop that by waiting a few months will be worth $140.

When he gets into the hands of the automobile salesman, and is bombarded with high pressure sales talk, he is probably fleeced and gets ten or twenty dollars worth of transportation for his $40 cash, another loss. Or he signs a contract which he does not understand. He does not read it and having no idea of the legal nature of the instrument, he gets into an impossible situation whereby in a few weeks or months, his car is repossessed and he has gone completely broke.

In the meantime, he has had use of a new instrument for which he is not prepared to make a cultural adjustment. He is not like the white farmer who has bought the car solely so that he may deliver his milk or vegetables to the city market or transport his feed and other supplies from the mill. No. He is like the white city brother who has bought the car to have the thrill of speed, to strut before his friends and enemies, and dizzily "go places."

Now, instead of staying at home and tending to his business, he enjoys a spree of visiting and wanderlust. Like so many of the youngsters whose conduct we bemoan today, he is able to travel far enough to have unrestricted access to gambling, liquor, and other demoralizations which we decry, whether white man or redskin.

I ask you, in all sincerity, what can education do on such problems? What can be our best educational approach? Will just teaching him how to count the number of cents in a dollar, or how to add and subtract, be of much real help to him? Will just teaching him the number combinations and mathematical formulas of rates of interest, such as has characterized our traditional organization of subject matter help him to solve his real problem?

To detail possible lines of attack and teaching techniques to deal with such realistic and crucial problems is not possible here. In general, the educational program must somehow, somewhere give the student actual experiences in understanding himself, in evaluating immediate versus postponed pleasures, in understanding and withstanding sales pressures, and in thinking through the social and economic adjustments to new conditions.

The illustration I have cited is not nearly as basic or complicated as many others facing us today in such things as land use, soil conservation, commercial agriculture, and a demoralized industrial production. The automobile is only symbolic but clearly shows the variables facing curriculum workers today.

Hence, with our increased knowledge of psychology, sociology, and the like, and with our increased awareness of the nature of our world today, at least three different roads face mathematical educators. First, they may continue to teach the highly compartmentalized, highly distilled sort of mathematics which we have so long known. Second they may reorganize the subjects of instruction in terms of the problems of our times. What these so-called subjects might best be, I do not know, and it is doubtful if, as a society, we ourselves know. They might be, I presume, such things as land use in the northeast, land use in the southwest, co-operatives, management of retail business, democracy, recreation, transportation, or a host of other possibilities. Some schools have already begun experimenting in such a form of organization, and this experimentation has many merits.

Third, and this seems most possible for mathematics teachers already in service, they may broaden the scope of their work so as to give children actual experience, either directly or vicariously, in the economic, psychological, and cultural settings

in which their quantitative thinking must operate.

At this point may I refer briefly to the work which George Boyce and I have done in the exploration of these implications. We personally feel that ours has been but a minor contribution to the total field and that our work has gone a very small way in the direction which the later twentieth century mathematical instruction must eventually go. Unfortunately, there are few other concrete illustrations of the desired pattern to which we can refer, and the paramount need today is for a host of experimenters and new publications dedicated to the new problems which inescapably surround us.

Our own work has particularly emphasized the possibilities of quantitative thinking in socio-economic problems because it is in this field that so little has been done. However, a course in general mathematics that emphasizes these socio-economic problems in a realistic way would make a tremendous contribution to the intelligent thinking of all groups, the mathematics specialist as well as the general consumer. Such a course would by no means imply that those students training for technical work should abandon their study of higher mathematics. It would simply represent an attempt to inject mathematical thinking into experiences and relationships which none of us can avoid.

The Thirteenth Yearbook of The National Council of Teachers of Mathematics by Dr. Fawcett indicates this new trend. His application of logic to non-geometric as well as geometric situations typifies the new spirit in which all of our mathematics courses need to be recast.

One of our biggest handicaps has been the failure of teachers to start out with a living problem—not simply a word problem that calls for an answer which is known in advance, but one which the child or growing adult faces in real life today. Obviously, in any realistic study of *life* problems, factors and variables which are non-mathematical must be recognized and dealt with, even where the purely mathematical possibilities are large. To abstract the mathematical content is to remove the problem from its realistic setting and rob it of vital significance. Unfortunately, when the mathematics teacher looks at a description of a realistic problem and is confronted with the non-mathematical elements, her immediate question is likely to be, "But where is the mathematics?" "This is social studies of physics or economics or something else."

Being this, it is viewed with alarm. The question that should be asked is "What does the child today really need?" This is the vital question in modern curriculum construction which is all too infrequently raised.

To summarize, in exploring the possibilities of mathematics for general education, no modern worker can continue to ignore the recent findings in other fields such as general psychology, mental hygiene, social philosophy, and the like. You cannot properly be satisfied with just a general tinkering and repackaging of the old product, even though it may be labeled under functional titles, if you are to make a fundamental attack on the problem of mathematics in general education. You cannot afford to be satisfied by problems which are set to give illusive practicality to theoretical mathematics while they actually fall outside the functional experience of the average individual.

Modern children are no less willing to work than were their predecessors in our schools. Their brains are no less active and keen, but they are much more aware of the problems of the world. And sensing the vitality of the need, they are impatient to be up and doing. They resent the respect of the older pedagogue for lifeless, purposeless, intellectual grind.

Mathematics must cease to be the toy of an intellectual aristocracy only, if mathematics is to assume its rightful role as a powerful tool to the great mass of society. And this means a fundamental new attack must be made with which I believe this organization is thoroughly concerned.

The Relation Between Secondary Mathematics and Physics and Chemistry

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The recognition of the importance of guidance in public education has brought about much study on the problems of the proper educational advising of our young people during the period preceding their entrance into a vocation or in succeeding educational activities. Intelligence tests have proved to be of value in this problem, especially if other facts are included as determining factors. There is so much material available on this subject that no special references need be made.

A further step in the educational guidance of pupils is to advise them regarding the selection of major courses of study, as contrasted with individual subjects. As the various subjects differ among themselves in the abilities which they demand, so do groups of subjects. The usual example given is the contrast in abilities required between the vocational and the academic groups.

Motivation and student interest have been found to be a large factor in school success. Any sort of advice or counsel which may take the student into the fullest confidence surely has much merit. It is not necessary to show an immediate practical use for everything to be learned, but wherever such reasons can be given they are desirable. It has been pointed out that a student "will work hard on subjects supposed to have a bearing upon preparation for the calling of his choice." A good way to motivate any learning is to bring the pupil to want to learn, and to make the results of good study desirable.

The purpose of this study is to try to determine some of the relations of success in algebra and geometry to success in physics and chemistry in high school so that the results may be of use as a motivation for the former, and for guidance in the latter. A question that often arises in first year algebra is, "Why study these subjects?" It has been the observation of the writer that the question is answered much more satisfactorily, and more interest is aroused and sustained, if it is pointed out that creditable work in these subjects will probably aid the student in mastering physics and chemistry or some other subject. The results of a study based upon the records of high school students themselves may be put in terms which the younger student can understand. A study in grades has limitations; a student, nevertheless, receives grades which are very important to him. The counselor's advice is conditioned largely by the scholastic record of the student. The method in this investigation has been, by a use of grades, to test the truth of the statement that creditable work in mathematics aids in mastering physics and chemistry.

The data for this study are based on the records of graduates of a single high school. Final semester grades were recorded in each of the following subjects: first year algebra, plane geometry, United States history, physics, and chemistry. The I.Q. as recorded on the permanent record was noted also. The grading system is on the basis of a five point scale.

This study is based upon coefficients of correlation between final grades in first year algebra and plane geometry, and physics and chemistry. History is used as one means of comparison. It was chosen because every student had taken it, and at approximately the same time that physics and chemistry were taken. The coefficients of correlation were derived by use of the product moment method and are known as Pearson coefficients of correlation. The coefficient may be thought of as a measure of the degree of correlation or correspondence between two sets of

grades. The actual computation follows the form given by Rugg.[1] The reliability of each of the coefficients of correlation was determined by finding the probable error.[2] The extreme infrequency with which the two lowest grades, "4" and "5" were given seemed to give too much weight to the grades of "5." Experiment seemed to indicate that these grades should be included in a single group in the computation of the correlation coefficient in this case, so this practice was followed.

No extensive study was made involving the I.Q.'s. They were, however, correlated with physics, chemistry, and history. They were weighted so as to be comparable with the grades, taking into account the half steps obtained when semester grades were averaged; for example, grades of "2" and "3" average "2.5."

Tables of Coefficients

In these tables of coefficients and hereafter the accepted symbol r will be used for the term, coefficient of correlation. *P. E.* will represent Probable Error and N the number of cases.

Table I

Correlations between Subjects

Subject	Subject	N	r	P.E.
Geometry	Chemistry	304	.6947	.022
Geometry	Physics	209	.6879	.025
Geometry	History	411	.5621	.023
Elem. Algebra	Chemistry	323	.5954	.024
Elem. Algebra	Physics	213	.4878	.035
Elem. Algebra	History	425	.5502	.023
I.Q.	Chemistry	294	.5806	.026
I.Q.	Physics	201	.4471	.038
I.Q.	History	413	.5565	.023
Elem. Algebra	Geometry	408	.6658	.019

Table II

Correlations between Semesters Same Subject

Subject	N	r	P.E.
Chemistry	293	.7959	.014
History	411	.6935	.017
Geometry	408	.6701	.018
Physics	181	.6061	.032
Algebra	422	.5923	.021
Average		.6716	

[1] H. O. Rugg, *Statistical Methods Applied to Education*, Houghton Mifflin Company, 1917. Chap. IX.

[2] Ibid., p. 272.

Before proceeding with specific comparisons, the reliability of the r's should be tested. The lowest limit usually given requires the r to be at least three times the P.E. in order to be considered reliable. More conservative practice indicates that r should be four or five times P.E. After applying this test it is seen that all of the r's are valid from this standpoint. A basis for determining high or low correlation is necessary. Merely setting some values of r as high or low does not seem as significant as a standard in terms of related data. The measure used here will be a comparison with correlations obtained for successive grades by semesters in the same subject. Thus first semester geometry is a prerequisite for second semester geometry, and first semester algebra for second semester algebra. This follows a recommendation made by Moore of the University of Cincinnati as a result of a study of correlations where high school grades were used. An average of the r's obtained (Table II) for successive grades in the same subjects gives us an r of .6716. It should be noted that the extremely high r of chemistry-chemistry (.7959) raises the average considerably.

An examination of Table I indicates a positive relation between success in geometry, and physics and chemistry. The r's meet the high standard of the average r for successive grades in the same subject. Comparison with the geometry-history r, further emphasizes the close relation between good work in geometry, and in chemistry and physics. When the probable errors are taken into account, it is seen that the algebra and I.Q. r's practically parallel each other in value. There is evidence that quality of work in first year algebra, as measured by final grades, accompanies quality of work in chemistry, physics, and history. The chemistry and history values are about equal, while the physics r is lower; but they are all high enough to indicate a positive relation. Geometry seems to predict much better the special abilities needed for physics and

chemistry, whereas algebra and the I.Q. select more general abilities as evidenced by the fact that the history r's for algebra and for the I.Q.'s are practically as high as those for physics. Geometry, algebra, and the I.Q. appear to be of equal value in predicting success in history. One other coefficient very worthy of note, however, is the high correlation between algebra and geometry. It compares favorably, also, with the high average r for successive grades in the same subject. Another thing to consider before final conclusions are drawn is that causal relationships may not be assumed. Good work in mathematics has a decided tendency to be followed by good work in chemistry. Success in chemistry may or may not be a result of good work in mathematics. The high correlation does not permit us to assume a cause.

The following conclusions seem to be justified:

1. That creditable work in mathematics will very probably aid the student in mastering physics and chemistry, is one valid answer to the question, "why study first year algebra and plane geometry?"
2. Elementary algebra and plane geometry may be motivated by suggesting a relatively immediate need for them. The relation between success in algebra and geometry is high.
3. The practical work of mathematics in high school should include physics and chemistry situations and problems, thus aiming at a transfer of very broad practical values as well as cultural and disciplinary values.
4. Mathematics grades offer a moderately dependable basis for guidance and counseling as far as work in physics and chemistry are concerned. The counselor and student can discuss grades more frankly and understandingly than I.Q.'s.
5. Geometry is the best single item to use in predicting success in physics and chemistry.

A former more comprehensive study (Master's Thesis, Stanford University, 1929) was made by the writer with a somewhat larger number of students from a different high school. The conclusions are in accord with those of the present investigation, especially those concerning geometry and chemistry. The results are not in disagreement as far as physics is concerned, but they are not comparable.

The relation of success in mathematics to success in physics and chemistry is very real and positive. The student is entitled to the feeling that his mathematics has a part to play in the success of the actual high school work he is now doing, rather than aiming too much at distant values and preparing for life. Mathematics has a definitely useful part to play in his high school life.

Classroom Practices of High School Teachers of Mathematics

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Sources of Data

The basic data of this investigation were obtained by means of a check list sent to approximately three hundred teachers of mathematics in Minnesota. From them 204 usable replies were received. Of these 25 taught in three-year senior high schools, 116 in conventional four-year schools, and 63 in junior high schools. The scope of the check list and the nature of the type of response requested may be gathered from Table I.

Findings and Conclusions

A summary of the findings is given in Table I. Among the more interesting conclusions, the following may be mentioned:

1. The comparatively small percentage regularly or frequently employing "telling" to introduce new topics.
2. The great extent to which collateral references are used.
3. The great extent to which the Morrison or some other unit plan is employed.
4. The large percentage of teachers giving no answer to the items related to the use of visual aids.
5. The great extent to which diagnostic and remedial procedures have come to be used.
6. The comparatively small percentage of teachers who do not use various appropriate devices for training pupils in methods of study.
7. The extent to which the flexible plan of dividing the class period time between recitation and supervised study has come to be used.

Variation of Practice with Size of School

The 145 replies received from four-year or senior high school teachers were equally divided among schools with enrollments of 82 or less, schools with enrollments of from 83 to 175, and schools with enrollments of from 180 to 3,370 (three replies did not state size of school). A few interesting differences in practice may be noted. In general there were no characteristic differences among the practices of the three groups of teachers.

On the staffs of the larger schools will be found a larger percentage of teachers who

1. Never use the lecture method to summarize a section of work or to explain visual material.
2. Never give short tests on the talks or lectures they give.
3. Never check required collateral reading by means of tests.
4. Regularly have students assist in planning attack on procedures in solving problems.
5. Regularly give practice in class on the use of study devices.

Variation of Teaching Practice with Amount of Teaching Experience

The teachers' classroom procedures in the four-year and the three-year senior high school groups were studied by groups in four different classifications:

1. Those with no more than two years of experience.
2. Those with three to five years of experience.
3. Those with six to fifteen years of experience.
4. Those with more than fifteen years of experience.

Certain characteristic trends were noted. The least experienced groups include a much higher percentage of teachers who

1. Never have individual reports on collateral reading.

TABLE I
Percentage of 204 High School Teachers of Mathematics Employing Certain Classroom Procedures

Procedures Reported Employed	Per Cent of Teachers Reporting Frequencies				Not Reporting
	Never	Occasionally	Frequently	Regularly	
1. Telling or lecture method for					
a. Introducing new topic	8.3	5.5	232.4	17.2	16.7
b. Summarizing a section of work	20.1	34.8	24.0	11.3	9.8
c. Introducing supplementary material	10.3	32.4	35.3	8.3	13.7
d. Explaining visual material	14.2	27.9	31.9	8.3	17.6
e. Arousing interest or appreciation	7.4	31.4	37.7	11.3	12.3
f. Aiding in mastery of difficult subject matter	8.8	17.2	36.8	28.9	8.3
2. Giving short tests on such talks	32.4	25.5	20.6	4.4	17.2
3. Assign collateral reading	48.0	20.6	8.8	4.4	18.1
4. Require collateral reading	55.9	12.7	7.8	3.9	19.6
5. Give suggestions for unassigned collateral reading	40.7	27.0	10.3	3.4	18.6
6. Check required collateral reading by written tests	56.9	12.7	5.9	2.9	21.6
7. Have notebooks or other reports on collateral reading handed in	56.4	8.3	4.4	6.4	24.5
8. Have individual oral reports on collateral reading given	46.6	18.1	10.8	2.9	21.6
9. Instead, occasionally question individuals on their investigations	36.3	24.5	7.4	1.5	30.4
10. Use problems involving several recitations	24.0	19.1	26.5	16.2	14.2
11. Set up problems in daily assignments	4.4	15.2	43.1	22.5	14.7
12. Attempt to organize subject matter psychologically instead of logically	22.5	17.6	29.9	15.2	14.7
13. Have students work out projects (set up radio sets, make scrap books, etc.)	30.9	27.0	13.2	5.4	23.5
14. Have students work out projects in groups	44.1	27.0	9.8	1.5	17.6
15. Have students work out projects as individuals	31.4	29.9	14.7	6.9	17.2
16. Have students assist in planning problems for class	37.7	33.8	9.3	2.5	16.7
17. Have students assist in planning attack or procedures in solving problems set up	16.7	21.1	27.0	17.6	17.6
18. Use Morrison Unit Plan (test, teach, retest, remedial teaching)	26.5	17.6	24.0	19.6	12.3
19. Use any form of contract plan	47.5	24.5	11.8	6.9	9.3
20. Use other types of large assignments	39.2	25.0	12.3	3.9	19.6
21. Use large unit assignment, supplemented almost daily by suggestions	21.1	16.2	13.7	13.7	35.3
22. Make differentiated or three level assignments	37.3	23.0	19.6	6.4	13.7
23. Use work books	31.9	13.7	14.7	33.8	5.9
24. Have students hand in papers or problems on daily assignments	5.4	5.4	25.0	61.8	2.5
25. Use socialized recitation	23.5	14.7	13.7	6.4	41.7
a. Use that of formal type (class organized as city council)	36.8	2.0	1.5	1.5	58.3
b. Use informal or discussion group type	7.8	23.5	31.9	15.2	21.6
26. Use question and answer recitation	9.8	30.4	41.2	12.3	6.4
27. Use topical outline	23.5	27.0	17.6	5.9	26.0
28. Allow students to keep their text books open during discussion	28.9	27.5	23.5	13.7	6.4
29. Employ visual instruction in any form					
a. Use slides	34.8	3.4	1.0	0.5	60.3
b. Use motion pictures	42.2	1.5	—	—	56.4
c. Use the stereoscope	42.6	1.0	—	—	56.4
d. Use maps or charts	23.5	14.2	17.6	5.9	38.7
e. Use pictures	18.6	14.2	22.1	4.4	40.7
f. Use a bulletin board	16.7	13.2	18.6	11.8	39.7
g. Use excursions or field trips	36.3	11.3	2.9	2.0	47.5
h. Use dramatization	39.7	5.9	1.5	.5	52.5
i. Use demonstrations	11.3	9.3	21.6	26.5	31.4
j. Use some other visual aid (please name)	6.4	1.0	5.4	3.4	83.8
30. Use standardized tests	21.6	20.6	29.4	14.7	13.7
31. Give pretests on sections or units of work	43.6	23.0	15.2	5.4	12.7
32. Give prognostic tests (measure capacity for a specific school subject)	59.8	15.2	7.8	2.9	14.2
33. Use diagnostic tests (to get at where and why students are weak)	9.8	20.6	39.7	25.5	4.4

TABLE I—*Continued*

Procedures Reported Employed	Per Cent of Teachers Reporting Frequencies				Not Reporting
	Never	Occasionally	Frequently	Regularly	
34. Use remedial teaching based on such tests	7.8	15.2	33.8	34.8	8.3
35. Give students instruction on how to study	1.0	24.5	42.6	41.7	1.0
36. Give lists of rules on how to study	31.9	27.4	14.7	17.2	8.8
37. Give practice in class on the use of study devices	13.2	29.9	29.9	15.7	11.3
38. Give drill and tests in the reading of assigned subject matter	16.2	26.0	28.4	13.7	15.7
39. Give special instruction and practice in skimming type of reading	53.4	9.8	6.9	1.0	28.9
40. Give special instruction and practice in study type of reading	25.5	19.6	20.6	6.4	27.9
41. Teach students how to use table of contents	18.1	27.0	23.5	22.1	9.3
42. Teach the use of the index	19.1	26.5	21.6	26.5	6.4
43. Teach students the purpose of footnotes	19.6	25.5	21.6	18.6	14.7
44. Teach the purpose and the use of bibliography	43.1	17.2	8.3	6.9	24.5
45. Give practice in finding answers to a set of questions	15.7	17.2	25.5	19.1	22.5
46. Give practice in formulating questions on a section of subject matter	23.0	26.5	18.6	8.3	23.5
47. Give students special attention in regard to how to take notes	40.7	17.6	12.3	4.9	24.5
48. Teach students how to outline	38.7	18.1	11.8	5.4	26.0
49. Give more practice to classes with low ability in study methods	17.2	12.7	28.4	17.2	24.5
50. Use supervised study time to aid students who study individually	6.4	13.7	27.0	44.1	8.8
51. Use supervised study time to work with the class as a group	11.8	27.0	27.0	22.5	11.8
			Yes	*No.*	
52. Have definitely divided period for supervised study			21.6	70.6	7.8
53. Use flexible method of dividing period for supervised study			74.5	16.7	8.8
54. Have definite plan of individual instruction with little or no group activity (Dalton or Winnetka Plan)			12.3	74.5	13.2
55. Use homogeneous grouping (teaching sections of all superior, all normal, or all dull students)			20.6	69.1	10.3
56. In using homogeneous grouping, do plans provide for material differences in subject matter and methods of instruction?			21.6	26.0	52.5
57. Is special work in how to study given?			2.5	88.7	8.8

2. Never have students assist in planning problems for class or have them assist in planning attack or procedure in solving problems set up.
3. Never use the Morrison large unit plan.
4. Never use the formal type of socialized recitation.
5. Never allow pupils to keep texts open during discussions.
6. Never employ field trips.
7. Never give practice in class in using study devices.
8. Never give drill and tests in reading of subject matter.
9. Never give special instruction and practice in reading.
10. Never give special attention to teaching students how to take notes.
11. Regularly use supervised study time to work with class as a group.

and a much smaller percentage who

1. Never have students work out projects.
2. Regularly use diagnostic tests and remedial teaching.
3. Regularly give students instruction on how to study.

The more experienced group includes a much higher percentage of teachers who

1. Attempt to organize subject matter psychologically instead of logically.
2. Regularly have students assist in

planning attack or procedure in solving problems.
3. Regularly give drill and tests in reading of subject matter.
4. Regularly give practice in study methods to classes with low ability.

and a smaller percentage who

1. Never use the Morrison large unit plan.
2. Never use standard tests.
3. Never give drill and tests in reading of subject matter.
4. Regularly use supervised study time to aid students who study individually.

Variation in Classroom Practice with Number of Years of Education Beyond High School

Of the senior and four-year high school teachers 52 had had only four years of training beyond high school and 34 had had more than four years. There were few characteristic differences in teaching practice between these two groups. Among them were the following:

1. Those with more training never had their pupils work out projects.
2. There was a tendency for those with more training never to use a work book and those with only four years to use a work book regularly.
3. Among those with but four years of training there was a larger percentage who never allow students to keep books open during discussion.
4. Those with less training tended more to use group supervised study regularly.
5. Those with less training more uniformly employed the definitely divided period for supervised study.

Variation in Classroom Practice with Number and Recency of Courses in Education

The replies were sorted into three groups on the basis of the number of courses taken in education. No characteristic differences were found among the teaching practices of these three groups.

There were more noticeable differences between the practices of the 57 teachers who reported that their last course in education was previous to 1931 and the practices of the 68 who reported their last course as taken in 1931 or later.

Those who had taken their last course in education more recently included a larger percentage who

1. Never had note books or other reports on collateral reading handed in.
2. Never used any form of contract plan.
3. Never used the formal type of socialized recitation.
4. Never used various types of visual aids.
5. Never used standard or "pre" tests.
6. Never employed the various devices for teaching pupils how to study.

Those who had taken their last course in education prior to 1931 included a larger percentage who

1. Regularly have pupils assist in planning attack or procedure in solving the problems set up.
2. Regularly employ some of the devices for training their pupils in methods of study.
3. Employ the flexible method of dividing the class period for supervised study.
4. Plan material differences in subject matter for homogeneous groups.

Cautions in Interpretating Data

One should be quite slow in attributing types of practice or differences in practice to size of school, amount of experience of the teacher, or any of the other factors discussed in this report. In all studies of this type it is probable that each of these factors is conditioned by one or more of the others, and possibly by still others not included in the study. For example, the teachers of greater experience are to a considerable extent also the teachers in the larger schools, and the teachers with their last course in education in 1931 or later are most probably disproportionately among those of least experience and in the smaller schools.

◆ THE ART OF TEACHING ◆

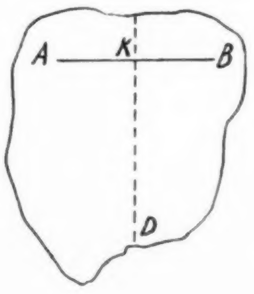
Paper Folding in Plane Geometry

By Sarah Louise Britton
Lincoln, Nebraska

From the practical minded sophomore comes the complaint: "Ruler and compass take too long. I'd never spend that much time constructing a perpendicular." Supplementary work in "easy" methods of construction appeals to this boy and challenges him to apply geometric principles to the solution of any construction problems which arise. The ingenuity thus developed may be a very valuable contribution of his geometry course to his life.

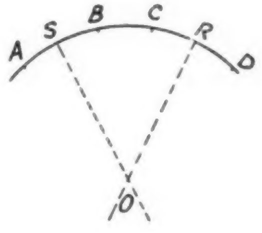
Most of the simple constructions of formal geometry can be carried out in paper folding. Bisecting an angle is a simple process of superposition. *To construct the perpendicular bisector of a segment, place one end point on the other and crease sharply along the resulting fold.*

The proof of such a construction can be quite as rigid as for the ruler and compass methods. Since AK has been made to coincide with KB they are equal. Likewise, angles DKA and DKB have been made to coincide and can therefore be shown to be right angles.

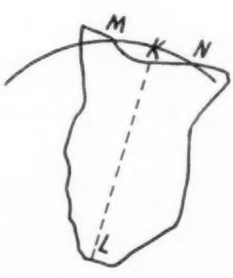


If the vertex angle of an isosceles triangle is given, the base angles may be found as follows: *Lay the straight edge of a paper along one side of the given angle. Fold back an angle equal to the given one. Bisect the remainder by a single fold.*

Given: an arc of a circle. **Required:** to find the center and the radiu. **Construction:** *Fold the paper so that any point, A, on the circle falls on any other point, B. Crease along the fold SO. Use any other two points of the circle, as C and D, to obtain another fold which will intersect with the first.* **Statement:** O is the center of the circle. **Proof:** As shown above, SO and RO are the perpendicular bisectors of two chords, AB and CD. The remainder of the proof is regulation geometry.



Suppose the arc is on a rigid body and cannot be folded. Tear off a piece of paper. Any shape will do. *Locate two points on the circle, as M and N. Mark them on the paper. When this is folded so that these points coincide, the line KL forms the*



perpendicular bisector of MN. Two other points may be treated in the same manner, and the center determined.

Tearing the paper instead of cutting it not only obviates the necessity for tools but offers opportunity for emphasizing geometric principles. Two points determine a straight line, and a better straight line can be obtained by folding through two points than but cutting. Likewise it is made clear that the points on the circle, rather than the chords joining them, are the essential elements.

After a few suggestions such as the above, students readily respond with problems and constructions of their own. A great deal of interest may be created and habits of ingenuity developed. The student is led to turn to his geometry as an aid in solving problems which arise, instead of shutting it up inside of a book at four o'clock each day.

EDITORIALS

Professor Breslich's Retirement

PROFESSOR E. R. BRESLICH, who was elected second vice-president of the National Council of Teachers of Mathematics at its annual meeting in Cleveland recently, is retiring this year from active service in the Department of Education at the University of Chicago. He was honored by the Council at the annual dinner by some remarks from the Council through a former Council president, J. P. Everett, as well as by his election to the vice-presidency.

It is fitting that the Council should thus honor a man who has given so much of his time and energy to the cause of mathematics. However, it is hard for those who know him to believe that he is ready to retire. It is probable that he will be active in the cause of mathematics for years to come and THE MATHEMATICS TEACHER takes this opportunity to wish him and Mrs. Breslich many more years of useful service and a happy life.

Professor Breslich was born on August 31, 1874, in East Prussia, Germany, one of a family of seven. He entered the Gymnasium at the age of nine, his favorite subjects being languages and mathematics. At the age of seventeen, one year before graduation, he spent one year visiting in the United States and did not return to Germany.

He first entered business here, but soon decided to fit himself for a professional career. So he went to what is now known as Baldwin-Wallace College where he received the B.A. degree in 1898. He received the M.A. degree from the University of Chicago in 1900, and also his Ph.D. degree there in 1926.

He taught first in Bradley Polytechnic Institute in Peoria, Illinois, and in 1904 went to teach mathematics in the University of Chicago High School, where he has for years been Head of the Department of Mathematics and Professor of Mathematics in the Department of Education.

For years he was associated with Professor G. W. Myers and a group of other teachers in working out courses in correlated mathematics which have since been revised and published by Professor Breslich.

Those who have been fortunate to work with Professor Breslich have found him to be a real supervisor whose own teaching was an excellent example to all those associated with him of how pupils should be taught.

Professor Breslich's leisure time activities are many, but he prefers traveling (by automobile), reading, and writing. He has written a three-volume series of books on the *Teaching, Technique, and Administration of Mathematics in Secondary Schools.*

Professor Breslich has four children, all graduates of the University of Chicago.

W.D.R.

Three Important New Books

THERE ARE three new books which all mathematics teachers will want to read. The first two are small books, one by Professor Bode on *Progressive Education at the Cross Roads* and another by Professor Dewey on *Education and Experience*. The first is published by Newson and Company, New York, and the second by Macmillan, New York. Whether one belongs to the so-called Progressive group or is found among the Essentialists, one can profit from a reading of both books. The Progressive will find a warning about going too far to the extreme in progressive tendencies without any well-planned organization of teaching materials, and the more conservative teachers will find some consolation for not having gone too far to to the left. However, no conservatively minded teacher should fall back upon the arguments set forth in these books as a reason for maintaining the *status quo*. The truth is that the best plan for our future course lies in steering our curriculum in mathematics somewhere between the more extreme views of the left and right.

The third book referred to above is a *Report of the Consultative Committee on*

Secondary Education with Special Reference to Grammar Schools and Technical High Schools and is published by His Majesty's Stationery Office in London. It can be obtained postpaid for $1.05 from the British Library of Information, 50 Rockefeller Plaza, New York City.

The main body of this report will be of some interest to mathematics teachers in this country, but its main interest for us lies in Appendices II to V, inclusive. Appendix II is a "Note by the Secretary on the Development of the Conception of General Liberal Education"; Appendix III is a "Memorandum on the Secondary Curriculum" by Dr. I. L. Kandel of Teachers College, Columbia University; Appendix IV is a "Historical Note on Faculty Psychology" by Dr. Cyril Burt, Professor of Psychology, University College, London; and Appendix V is a "Memorandum on the Cognitive Aspects of Transfer of Training" by Dr. H. R. Hamley, Professor of Education and Acting Director of The Institute of Education at the University of London. Dr. Hamley is also the author of the Ninth Yearbook of the National Council of Teachers of Mathematics on *Relational and Functional Thinking in Mathematics.*

W.D.R.

Mathematics in California High Schools

In an article by Frank B. Lindsay, Assistant Chief, Division of Secondary Education in California, in *California Schools* for April 1939, we find the following statement:

> Replies from 324 public high school principals establish that more than half of these institutions have moved algebra from the ninth to the tenth grade. Plane geometry is an eleventh-year subject in more than a third of these schools; in some it is even postponed to the twelfth year. By review and remedial procedures 54 per cent of the schools attempt to secure mastery of fundamental arithmetic processes. Many programs of modified mathematics are already in operation in California public high schools. Principals and teachers of mathematics have joined efforts to reduce failures in algebra and geometry. Shifting algebra and geometry to the upper grades signified general acceptance of the statement that "It should be recognized by teachers and counselors that plane geometry is often attempted by pupils before they are mature enough to appreciate the logical development of the subject."

It is strange that so many people in this country think that the solution to the algebra and geometry situation consists in postponing the teaching of them to later grades. The fact is that the study of mathematics should be begun earlier and extended throughout the secondary school period, because in this complex civilization which we are now entering we are going to need to know more mathematics, not less.

It should be said, however, that the mathematics to be taught should be entirely reorganized for teaching purposes. Our failure to do this satisfactorily and to teach the subject better is the main cause of our present trouble. Many children will need to learn all the mathematics possible if they are to be leaders, and they will be handicapped if they do not start their mathematics early. Moreover, the introduction of social mathematics will not necessarily solve the problem. Much of the arithmetic that is being introduced is harder for the pupils than the algebra it displaces.

It is most unfortunate that the idea has gone around that general mathematics is a "dumbbell course." The fact that it is more interesting to many slow pupils than algebra does not mean that it is worthless. The easy way to get rid of a subject that demands thinking for its assimilation is to throw it out or postpone it, but in the case of mathematics this is sure to be the short-sighted way to handle the situation. Those who really wish to solve the problem should study the forthcoming Report of the Joint Commission on *The Place of Mathematics in Secondary Education* which will appear as the Fourteenth Yearbook of the National Council of Teachers of Mathematics.

W.D.R.

◆ IN OTHER PERIODICALS ◆

By Nathan Lazar
Alexander Hamilton High School, Brooklyn, New York

1. Addelston, Lorraine W., "Shall We Teach to Test—or Test to Teach?" *High Points*, Vol. 21, No. 3, March, 1939, pp. 38–43.

The continuation of an article that appeared in the same magazine in September, 1938. In this article the writer deals with the following topics:

a. How professional test-makers make a test;

b. Current trends in test making:

c. What the teacher can do, when making a test, to achieve the aims and objectives which are set forth as the fundamental justification for the teaching of mathematics.

Promise is given of a sequel to this article in which a report will be given of an analysis of a great number of midterm and endterm tests given in the high schools of New York City.

2. Baten, William Dowell, "Recent Interests in Probability and Mathematical Statistics." *Scripta Mathematica*, 5: 165–170, July, 1938.

In this paper the author gives a brief sketch of the development of probability and mathematical statistics during the past twenty years.

a. "Most of the universities and colleges examined have introduced probability and statistics into their curricula during the past two decades.

b. "Seven new journals have arisen during the past twenty years which are devoting their pages to these subjects entirely.

c. "During the past year many more articles on probability and statistics appeared in scientific journals of the world than in scientific journals twenty years ago."

d. A large majority of the books on these subjects have been written in the past two decades.

e. "Many other sciences are using the knowledge of these subjects to develop and perfect their work.

f. "One of our largest universities is allowing students in certain fields to replace one foreign language requirement by one year of mathematical statistics.

g. "Many students the world over are doing serious study in these fields of mathematics."

3. Bergen, M. C. "Comparison of Grades in College Trigonometry between Students Who Had the High School Course and Those Who Did Not." *School Science and Mathematics*, 39: 273–274. March, 1939.

The writer compared the grades in college trigonometry of 119 students who had the high school course with the grades of 122 students who did not have the high school course. He found that, on the whole, the difference is much less than might have been expected on *a priori* grounds.

4. Erskine, W. H., "The Contents of a Course in Algebra for Prospective Secondary School Teachers of Mathematics." *The American Mathematical Monthly*, 46: 32–35, January, 1939.

The author first points out the defects of the philosophy and contents of the course that is usually the terminal undergraduate contact with algebra for prospective secondary school teachers. He then gives the following outline of the topics which he believes should be included in such a course:

a. Division algorithms of Euclid. Unique factorization of natural numbers. The divisors of an integer.

b. Infinitude of primes. Conjectures on the distribution of primes.

c. Pythagorean numbers. Fermat's last theorem.

d. Concepts of "class," "belonging to a class," "correspondence between classes."

e. The class of positive integers and denumerable classes.

f. The concepts of "group," "ring," "field."

g. Algebraic integers. Failure of unique factorization rule.

h. Divisibility tests for rational integers. Congruences.

i. Scales of notation. The relation between the polynomial and the integer.

5. Huntington, Edward V., "The Duplicity of Logic." *Scripta Mathematica*, (a) 5: 149–157, July, 1938. (b) 5: 233–238, October, 1938.

The anomalous situation in which logic and mathematics find themselves has long been a

source of disturbance to students of these sciences. "On the one hand we used to say that anything proved by logic must be true. . . . On the other hand, we now say that logic does not deal with truth. The propositions supposed to be proved by logical reasoning are not propositions at all, but propositional functions, which as such are neither true nor false. In short nothing proved by logic can possibly be true! Here is indeed a double life. On the one hand logic is the official guarantor of truth. On the other hand logic has nothing to do with truth."

The author resolves this dilemma by means of very carefully chosen examples and by a clear exposition of the differences between "Pre-Mathematical Observations on Space" and a pure mathematical system. He also points out that the double life led by mathematics and logic "is a source of strength rather than of weakness, a gain rather than a loss."

6. Jeffrey, G. B. "Mathematics in School and University." *The Mathematical Gazette*, 23: 26–34, February, 1939.

Taking advantage of the roving nature of a presidential address, the writer makes many penetrating comments on a great number of topics.

"To the schoolboy the things of mathematics are simple ideas illustrated on every hand in his everyday life. Moreover the assertions of mathematics are susceptible of clear and concise statements. Hence it seems to me that mathematics gives us the opportunity of thinking clearly and in a simple way about some of the simplest notions that the human mind possesses. It is the superlative introduction to the art of clear thinking. . . . The boy who can make a calculation correctly in five minutes is better than the boy who takes fifteen minutes for the job, and the examination paper may rightly test both accuracy and speed. I do not think that the same is true of geometry to anything like the same extent. A rider [i.e., an "original"] is an exercise in clear thought and most of us find it difficult to think clearly if we are hurried and pressed for time. . . . We may justify our subjects on the ground of its utility. . . . We need not adopt a purely vocational view of education in order to justify our subject. . . . Simple three dimensional geometry should be given a larger place in our school work, for it is three dimensional space rather than two dimensional that we discover through our perceptions. One of the best ways of learning the simple geometry I have in mind is through handicraft. . . . Another ground for our faith in the value of our subject is to be found in what I believe to be a fact, namely, that a considerable proportion of our pupils enjoy mathematics. . . . There is a value in the discipline of relatively uncongenial subjects. We have to learn sooner or later that we cannot go through life devoting our whole time to enjoyable tasks."

7. Karpinski, Louis C., "A Problem of Presentation in Trigonometry." *National Mathematics Magazine*, 13: 240–241, February, 1939.

It is urged that in teaching trigonometry the expressions—side opposite, side adjacent, and hypotenuse—should be dropped, and be replaced by x, y, and r (or v). The primary basic definitions, formulas, and relationships would then be the following:

$$\text{Sin } \theta=\frac{y}{r};\ \cos\ \theta=\frac{x}{r};\ \tan\ \theta=\frac{\sin\theta}{\cos\theta}=\left[\frac{y}{x}\right];$$

$\sin^2\theta+\cos^2\theta=1;\ 1+\tan^2\theta=\sec^2\theta.$
The advantages to be derived from such an approach are indicated.

8. Miller, G. A., "A First Lesson in the History of Mathematics," *National Mathematics Magazine*, 13: 272–277, March, 1939.

The following are some of the primary facts of the history of mathematics: "Relatively few of the fundamental questions in which we are likely to become interested can be answered now. . . . There is no evidence which implies that any of the ancient peoples regarded the totality of the positive rational numbers as a connected system. . . . In general, historical conclusions should be regarded as working hypotheses only, and one should be continually ready to consider additional evidence. . . . The developments in elementary mathematics were affected much more by looking backward than by looking forward, in the sense that general rules and homogeneity were an outgrowth of special developments. These special developments were frequently inspired by the immediate needs. . . . It should be emphasized that mathematical history differs widely from mathematical tradition. Among the traditions which have recently been frequently stated is the one that the ancient Egyptians constructed right angles by means of a rope which knots separated by distances in the proportion of 3, 4, 5. This legend seems to have been started by the noted German mathematical historian Moritz Cantor (1829–1920) and it was due to a misinterpretation. . . . Many of the fundamental advances in mathematics were not premediated by men of unusual foresight, but resulted from the practices of many who improved a little here and a little there on the work of their predecessors without knowing that they were contributing thereby toward

the advancement of an important subject. . . . As far as available literary evidences furnish a basis of judgment, pure mathematics is as old as applied mathematics."

9. Nyberg, Joseph A., "Teaching Geometry and Logic." *School Science and Mathematics*, 39: 201–209, March, 1939.

After a brief discussion of various problems centering around the question of teaching logic through geometry, the author continues with a detailed exposition of the method he has used in the classroom for more than twenty years.

No teacher should fail to read this article, so replete with homely illustrations and classroom suggestions. It is, however, unfortunate, that despite the author's acquaintance with the literature on the subject, he still uses the terms "converse," "inverse," and "contrapositive" preceded by the definite article "the," as if theorems have but one converse, one inverse, and one contrapositive. Such is the case only when a theorem has but one condition in its hypothesis. The number of such geometric theorems is very small. The advantages to be derived from a multi-converse, multi-inverse, and multi-contrapositive approach have already been described elsewhere in great detail and will not be repeated here.

10. Read, Cecil B., "Vexing Minor Problems of the Mathematics Curriculum." *National Mathematics Magazine*, 13: 237–239, February, 1939.

A description of some problems encountered in the administration of the mathematics department of the Municipal University of Wichita.

What provision is to be made for those students who although possessing sufficient credit to enter college, lack certain mathematics courses prerequisite for further mathematical work? What shall be done with those students who *did* have the prerequisite course but lack the proper foundation for the advanced course? How can an instructor be sure that he has two equivalent forms of the same examination? What is the relative value of the work prepared outside the class? How large may a section be for effective teaching?

The tentative answers are given to some of the above questions.

11. Short, R. L., "Methods in Arithmetic and Algebra." *School Sciences and Mathematics*, 39: 239–250, March, 1939.

The author believes that mathematics becomes a great problem in the grade school and in the high school unless it is presented to the pupils on the basis of reasoning. Arithmetic is therefore developed so that it is understood and is interesting, thus doing away with unnecessary memorizing of meaningless rules and confusing directions. This article contains ten pages of practical, classroom examples and procedures to substitute reasoning for rules and understanding for memorizing.

12. Watson, E. E., "Our Geometric Environment." *School Science and Mathematics*, 39: 258–269, March, 1939.

This article contains the answer to many geometry teachers' prayers. It includes fascinating examples from every conceivable branch of science to illustrate common geometric forms.

The Educated Person Solves His Problems of Counting and Calculating

SOME acquaintance with numbers and skill in fundamental operations of addition, subtraction, multiplication, and division is an educational objective to be taken for granted. The skills to be taught in this field and the types of problems to which these skills are applied should be determined by the kinds of arithmetical calculations which the ordinary American citizen has occasion to make. Elaborate and helpful investigations have been made to bring these fundamental operations into a position of prominence and recent revision of the curriculum in many school systems has resulted in great improvement in arithmetic instruction. In addition to skill in mathematics there needs to be developed an appreciation of the cultural value of mathematics, and of its usefulness as a mode of thinking and as a means of interpreting world affairs.—*The Purposes of Education in American Democracy*, Educational Policies Commission.

Summer Meeting of the National Council of Teachers of Mathematics

San Francisco, California, July 3-5, 1939

N. E. A. Theme: Civic Responsibility

N. C. T. M. Theme: Teaching Mathematics to Meet Social Needs

Monday, July 3

3:00 P.M., Mission High School (tentative), San Francisco

JOINT MEETING with the Department of Secondary Education of the N. E. A.

Presiding: H. C. Christofferson, President N. C. T. M., Professor of Mathematics, Miami University, Oxford, Ohio

1. Address of Welcome: Vaughan MacCaughey, Editor, *Sierra Educational News*
2. "The Demand Which the Modern Elementary Curriculum Makes on Arithmetic"—Paul R. Hanna, Professor of Education, Stanford University
3. "The Contribution of Mathematics to General Education"—E. R. Hedrick, Vice-President and Provost, University of California, Los Angeles

Tuesday, July 4

2:00 P. M., High School of Commerce (tentative), San Francisco

ARITHMETIC IN A MODERN ELEMENTARY SCHOOL

Presiding: Helen Heffernan, Chief of the Division of Elementary Education, California State Department of Education, Sacramento

1. "Making Arithmetic Meaningful to Children"—E. A. Bond, Professor of Mathematics, Western Washington College of Education, Bellingham, Washington
2. Discussion

2:00 P.M., High School of Commerce (tentative), San Francisco

MATHEMATICS IN THE SENIOR HIGH SCHOOL

Presiding: H. C. Christofferson, President N. C. T. M.

1. "High School Mathematics in the University"—H. M. Bacon, Assistant Professor of Mathematics, Stanford University
2. "Shall We Accept the Two Years of Required Academic Mathematics as a Right, a Ritual, or a Challenge?"—Ruth Sumner, Teacher of Mathematics, Oakland High School, Oakland, California
3. "The Place of Mathematics in Secondary Education"—the Reverend William C. Cianera, S. J., Dean of the Faculties, University of Santa Clara
4. Discussion.

Wednesday, July 5

12:45 P.M., The International House (Piedmont Avenue and Bancroft Way), University of California, Berkeley

LUNCHEON WITH DISCUSSION GROUPS

Leaders to Be Announced

Reservations for this luncheon should be made with the chairman, Mrs. Ruth Sumner, 3000 Central Avenue, Alameda, California, as soon as possible. Price 88 cents, tax included.

3:00 P.M., Life Sciences Building, University of California, Berkeley, Room 2000

MATHEMATICS FOR THE MODERN JUNIOR HIGH SCHOOL

Presiding: Earl Murray, Assistant Director of Curriculum, Santa Barbara Schools, Teacher of Mathematics, Santa Barbara High School, Santa Barbara, California

Panel Discussion, members of the panel to be announced

3:00 P.M., Life Sciences Building, University of California, Berkeley, Room 2003

MATHEMATICS IN THE JUNIOR COLLEGE AND IN TEACHER TRAINING

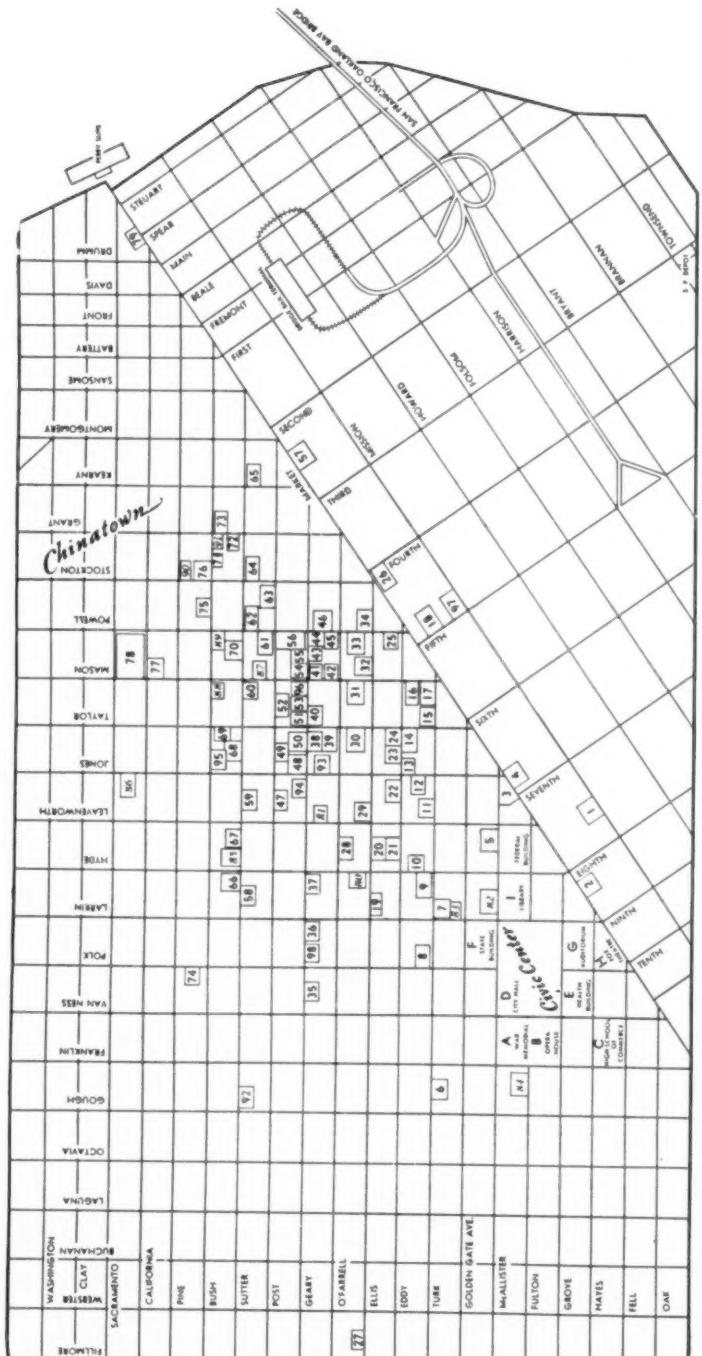
Presiding: H. F. Minssen, Vice President and Professor of Mathematics, San Jose State College, San Jose, California

1. "Coordination in the Teaching of Mathematics"—Sophia H. Levy, Assistant Professor of Mathematics, University of California, Berkeley
2. "Mathematics for a Liberal Education"—Arnold Dresden, Professor of Mathematics, Swarthmore College, Swarthmore, Pennsylvania

HEADQUARTERS of the National Council of Teachers of Mathematics, THE PALACE HOTEL, San Francisco

H. M. Bacon, Stanford University, Stanford University, California, Chairman Local Arrangements and Program Committee.

STREET PLAN OF SAN FRANCISCO



NEWS NOTES

The spring meeting of the Mathematics Section of the California Teachers Association, Bay Section, was held March 22, at the University of California. The speaker was Provost E. R. Hedrick of the University of California at Los Angeles. His topic was "Attitudes Toward Mathematical Teaching In Secondary Schools and Colleges." He said that the selection of the topics which we teach and our emphases upon them are determined by our attitudes. The oldest attitude is discipline. It is unnecessary to emphasize it too much. We should question the teaching of some moderns that anything hard is necessarily bad. On the other hand anything hard is not necessarily good. Judgment and intelligence must be exercised that we do not carry this attitude too far.

Transfer of training has been taken too much for granted. Experience shows that if we want transfer we must lead students by the hand to get it. Application must be direct. The frequently quoted "Mathematics has no applications in life," can be disproved on every hand. We are responsible for seeing that it is taught so the applications are noted. Such obvious cases as installment buying, compound interest, and rates on loans are enough to illustrate. The function concept or relation between quantities is another important attitude. And in analytic geometry in college your attitude may be on beautiful theorems in geometry or upon functional relationships in quantities which actually occur. If mathematics is not in touch with life, perhaps it is our fault. We can assist in integration and socialization in economics through topics of insurance, bond issues, pensions, and taxes. There would not have been so much voting on the Thirty Thursday Plan if people had known their mathematics. We must look carefully at our attitudes to be the best teachers of mathematics.

A large and enthusiastic group attended the meeting. Many distinguished teachers and administrators of the University were present also to honor Dr. Hedrick.

After the talk a short business meeting was held at which the officers for the following year were elected as follows: Advisory Chairman, Mrs. Beatrice Anderson, Alameda High School, Alameda; East Bay Section, Chairman, Mrs. Ruth Sumner, Oakland High School, Oakland; Secretary, Miss Helen Manny, Oakland High School, Oakland; West Bay Section, Chairman, Mr. Ivan C. Barker, Lowell High School, San Francisco; Secretary, Mr. Adolph Spiess, Mission High School, San Francisco. Announcement of the summer meeting of the National Council to be held July 3–5, jointly with the N.E.A. was made.

EMMA HESSE

The fall meeting of the Mathematics Section of the Bay Section, California Teachers Association, was held December 7, at the Westlake Junior High School in Oakland. The chairman, Mrs. Beatrice Anderson, called the meeting to order and introduced the speaker, Mr. Arthur W. England, actuary associated with Coates and Herfurth, Consulting Actuaries, San Francisco. Mr. England spoke on "Mathematics Underlying Life Insurance and Retirement Plans." His topic was timely, in consideration of currently interesting legislation so recently before the voters of California. It was an interesting exposition of socializing mathematics.

As usual, a large attendance greeted the speaker.

EMMA HESSE

The Association of Mathematics Teachers of New Jersey held its Sixty-fifth Regular Meeting at the State Teachers College, in Montclair, on March 4.

The program included demonstrations under the general direction of Dr. Foster E. Grossnickle, State Teachers College, Jersey City. Miss Elizabeth Shaner, Columbian School, East Orange, conducted the demonstration for the third grade; and Miss Elsie Mabee, State Teachers College, Jersey City, for the fifth grade.

"Drill in Fundamentals" was the theme of Dr. E. H. C. Hildebrandt's eighth grade demonstration. Dr. Hildebrandt's colleague at the State Teachers College, Montclair, Professor Howard F. Fehr, conducted the twelfth grade demonstration on "Method of Analysis."

Papers were read by Miss Mary Rogers, Junior High School, Westfield, on "Vitalizing Junior High School Mathematics for the Slow Learner," and by Dr. David R. Davis, State Teachers College, Montclair, on "Ways in Which a Knowledge of Higher Mathematics May Enrich the Teaching of Senior High School Mathematics."

Professor Maurice L. Hartung, University of Chicago, and Assistant Director of the Pro-

gressive Education Association, spoke at a general meeting on "Unifying Concepts in the Mathematics Curriculum."

The seventh annual Summer Conference of the Stanford University School of Education will be held from July 7 to 9, immediately following the meetings of the National Education Association in San Francisco.

"Educational Frontiers" is to be the central theme for discussion at this year's Conference. The topic will be approached from the point of view of, first, the social frontiers which are making new demands upon educational practice and thought; second, the psycho-biological frontiers where research is continually revealing new and educationally significant facts about the human organism; third, significant innovations on educational frontiers consonant with the social and biological frontiers; and fourth, an evaluation of the success of our culture in meeting the needs of youth.

Howard W. Odum of the Institute for Research in Social Science, University of North Carolina, will speak on *Social Frontiers* at the opening session of the Conference, Friday, July 7, at 10 o'clock. On Friday evening, Lewis M. Terman, Professor of Psychology at Stanford University, will speak on *New Evidence on the Nature of the Human Organism.* Jesse H. Newlon, Professor of Education at Teachers College, Columbia University, will discuss *Educational Frontiers* at the general session on Saturday evening; and John W. Studebaker, United States Commissioner of Education, will speak on the subject *Youth Challenges the Culture* at the final session on Sunday morning, July 9.

On Friday afternoon, Saturday morning, and Saturday afternoon there will be a series of forum sessions dealing with many aspects of the central theme and giving opportunity to attendants at the Conference to participate actively and critically in the discussions. Speakers at the forum sessions will be chosen from among educational leaders in all parts of the country.

The Conference is open to the public and will be of particular value to parents, teachers, educational administrators, adult education groups, and social groups which devote a part of their activity to educational work. Stanford University, located thirty-five miles from San Francisco, is easily accessible by train, bus, or automobile to those attending the N.E.A. meetings and the Golden Gate International Exposition. For the duration of the Conference, rooms will be available on the campus to persons who make reservations in advance.

For detailed information on the program, fees, accommodations, etc., write Stanford Education Conference, Stanford University.

"Whither Mathematics?" was the general theme of the joint luncheon of The Mathematics Chairmen's Association and the Association of Teachers of Mathematics of New York City on March 11. The sessions started at ten o'clock in the morning and included four panels organized as follows:

PANEL I

Ninth Year Mathematics. Chairman: Mr. Albert E. King, Principal Jr. H. S. 66. Problems of Teaching Mathematics in the Junior H. S., Mr. George L. Paley, Jr. H. S. 66; Mathematics for the Slow Pupil, Dr. Harry Eisner, Manual Training H. S., Mrs. Rose R. Turtz, Haaren H. S.; Vitalizing the Teaching of Certain Topics, Mr. Benjamin Braverman, High School of Commerce, Mr. Joseph P. McCormack, Theodore Roosevelt H. S.

PANEL II

Tendencies Toward Integration in Secondary Mathematics. Chairman: Miss Etta Greenberg, Wash. Irving H. S. History and Survey of the Integration Movement, Mr. Morris Hertzig, Haaren H. S.; Ninth and Tenth Year Comprehensive Course, Mr. Fred Schoenberg, Straubenmuller Textile H. S., Mr. George Ross, Thomas Jefferson H. S.; Eleventh and Twelfth Year Comprehensive Courses, Mr. Meyer Weiner, New Utrecht H. S., Mr. Samuel Atlwerger, High School of Science.

PANEL III

Conflicting Tendencies in the Teaching of Geometry. Chairman: Mr. Samuel Welkowitz, Franklin K. Lane H. S. Cartesian and Synthetic Proofs, Dr. Edna K. Lassar, Thomas Jefferson H. S.; Logic in Geometry, Mr. Mannis Charosh, New Utrecht H. S., Mr. Michael Katzoff, Grover Cleveland H. S.; Applications of Reasoning to Non-Mathematical Situations, Mr. Julius Hlavaty, High School of Science; Teaching and Testing Appreciations and Attitudes, Mr. Jack Deutsch, Thomas Jefferson H. S., Mr. Cyril Graze, High School of Commerce.

PANEL IV

Mathematics for All the Children. Chairman: Mr. Joseph B. Orleans. The Place of Remedial Reading, Mr. Will Scarlett, Technical Supervisor, Remedial Program, Miss Etta Waite, Girls Commercial H. S., Project of W. P. A.; How to Train Pupils to be Autodidactic, Mr. Clinton A. Bergstresser, Jamaica H. S.; Specialized Mathematics, Miss Alma Ekholm, Girls Commercial H. S., Mrs. Pearl G. Bennett, Brooklyn H. S. of Women's Garment Trades; Testing What All the Children Know of Mathematics, Mrs. Lorraine W. Addelston, P. S. 159, Manhattan.

NEW BOOKS

A Study of the Possibilities of Graphs as a Means of Instruction in the First Four Grades of the Elementary School. By Ruth G. Strickland. Bureau of Publications, Teachers College, Columbia University Contributions to Education, No. 745, 1938. 172 pp. Price, $1.60.

Teachers and educators concerned with materials and methods of elementary school instruction and writers and publishers of elementary school textbooks should acquaint themselves with this very practical study. As Dr. Strickland points out in her statement of the problem, graphs more and more are becoming "an integral part of the daily reading matter of the American public." Advertisers and others whose business it is to present information in a simple, dramatic, and effective manner have exploited graphs for their various purposes. Can they be exploited in the interest of the education of our four- to ten-year-old children; that is, in the first four grades?

Dr. Strickland puts the question in three parts:

1. What types of graphs are intelligible to children at the different levels from the first through the fourth grade?
2. What types of graphs can be understood at the mental age levels within each grade?
3. What minimum of instruction is necessary to make these graphs intelligible at the levels studied?

The details of the investigation of the problem, using 461 children in the first four grades can best be read in the report. The sponsorship and approval of Professor Helen Walker, of Teachers College, Columbia University, will be sufficient guarantee to many that the method, statistical treatment, and conclusions of the study may be accepted as sound.

That the use of graphs can be started much earlier in the schooling process than is generally thought practicable is warranted by the children's liking for and interest in the charts used in the study and a resonable measure of success in dealing with the charts. Even first-grade children were found to be capable of answering questions based on the unit pictograph and developmental picture chart.

Of special interest to the mathematics teacher is Dr. Strickland's observation that "The use of graphs in the social studies program would facilitate a combination of social studies and arithmetic that should add richness of content to the social studies program as well as growth in understanding and skill in quantitative thinking.

"There is an increasing tendency on the part of teachers to make the program of the first years in arithmetic concrete work with materials and content which develops vocabulary and concepts as a basis for later contact with abstract numbers and fundamental processes. It is quite possible that many of the quantitative concepts which should be taught in these early years can be strengthened through use of graphs if these are suited to the interests and abilities of the children."

A bibliography of nineteen items in the literature on graphs and picture statistics provides the reader with suggestions for further study. It seems as though, at this point, should also be mentioned Rudolf Modley's *How to Use Pictorial Statistics*, Harper, for the use of anyone eager to investigate further the possibilities in graphic teaching.

E.W.

Business Mathematics. Second edition. By Isaiah Leslie Miller and Clarence H. Richardson. D. Van Nostrand Company, 1939. 352 pp. Price, $3.75.

This is a second edition of a book, first published in February, 1935, to be used as a text for a one-year college course in the mathematics of modern business and finance.

The first nine of the thirty-one chapters are devoted to theoretical mathematics, algebra for most part. A year or a year and a half of algebra are supposed to precede the course. These preparatory theoretical chapters cover such topics are algebraic operations, linear equations in one unknown, binomial expansion, logarithms, quadratic equations, progressions, and functions.

It is interesting to see that in the "practical" field of business training such theoretical topics are specifically taught before the topics like annuities and valuation of bonds are touched on. Particularly in the chapter on function, it is satisfying for a mathematics teacher to see a topic like finding the equation of a line, given its slope and the coordinates of one point on the line. The student is expected to be able to write equations from data concerning cost, depreciation, and other situations in which variables operate.

The bulk of the book consists of a thorough exposition of the mathematics fundamental in simple interest and discount, compound interest and compound discount, annuities, sinking funds and amortization, depreciation, valuation of bonds, probability and its application in life insurance, and other problems in insurance.

The last pages contain useful tables of five and seven place logarithms, compound interest, and annuity and insurance facts.

E.W.

Plane Trigonometry. By George M. Hayes and Murray J. Leventhal. Globe Book Company, 1938. 248, pp. Price, $1.60.

The authors have justified this new trigonometry by their attempt to bridge the gap between educational theory and practice. Illustrative of this purpose is the easy, complete, step-by-step development of the subject. For instance, the first three trigonometric functions are defined, described, and worked with in simple exercises on pages 6 through 11. The reciprocal functions are not mentioned until page 12 after the student has had an opportunity to master and learn to think clearly about the first three.

Of value to teacher and class is the six-page section on "Approximate Numbers." The question "to how many places?" is a troublesome one in problems involving considerable decimal arithmetic, one many teachers have not thought through or taught consistently. The careful explanation here is convenient for reference, and although not so urgently necessary as the tables of functions and logarithms, it is worth having to insure accurate work and uniformity of results in the class.

The book may be used either in a high school or college course, although most college instructors would want a more comprehensive text. There are extra problems for the bright boys. Sections on De Moivre's Theorem and Polar Coordinates are last in the book. The authors suggest that these be omitted in a short course.

Several dated "Specimen Examinations" are given in the pages just before the Index. The editors might very well have indicated what testing or accrediting agency gave these examinations.

E.W.

Review Course in Algebra. By W. E. Sewell. D. C. Heath and Company, 1938. 145 pp. Price, $1.20.

As the title suggests, this is an algebra for those who have studied the subject but need to review it either in preparation for college or for use in business and the professions. Its explanations are complete enough so that a mature student who has forgotten much of algebra can regain mastery by himself. However, no one should make the mistake of trying to use it with beginners, mature or not. Definition and illustration of principles is insufficient for those to whom the ideas will be new.

Among the topics treated in the eleven chapters are: Exponents and Radicals, Algebraic Reductions, Linear Equations and Systems (including Determinants), Quadratic Equations and Systems, Functions and Graphs, Ratio, Proportion, Variation, Progressions, and the Binomial Theorem.

A final review section contains many problems typical of those appearing on college entrance examinations.

E.W.

Laboratory Experiments in Elementary Physics, to accompany Black and Davis' *Elementary Practical Physics.* By Newton Henry Black. The Macmillan Company, 1938, 263 pp. Price, $1.24.

This manual is important because it is designed to accompany a textbook which is widely used and highly regarded. The manual is recommended for both high school and college classes in beginning physics. Dr. Black, who is assistant professor of physics at Harvard, has turned out a thoroughly good laboratory guide, which covers the field, has good introductions and suggestions for each exercise, and clear, irreproachable diagrams.

It contains sixty-one experiments of which thirty starred ones are considered a basic minimum. The experiments are grouped under divisions: Mechanics, Heat, Magnetism and Electricity, Sound, Light, and Modern Physics. The division on Modern Physics contains one experiment entitled "Characteristics of a Vacuum Tube."

Many high school teachers would question the practicability of a laboratory manual like this one, size 5½ by 8½, cloth bound. Most students prefer the larger, paper covered, workbook type of manual with forms and tables set up for recording data directly in the book. This manual has models of forms for recording data which the student may copy in his own notebook, but he is not supposed to write in the manual, nor are the sample forms large enough for that purpose, should the student wish to do so.

College instructors, who sometimes wish to have their students work out their own forms for presenting data and results, may prefer this manual to the ones that are made to be written in.

E.W.

Enriched Teaching of Mathematics in the Junior and Senior High School. Revised edition. By Maxie Nave Woodring and Vera Sanford. Bureau of Publications, Teachers College, Columbia University, 1938. 133 pp. Price, $1.75.

Here is a revised edition of a book which, since its first publication in 1928, has contributed to energetic and inspired mathematics teaching at its best and has furnished many a lift to discouraged teachers and bored classes. The discovery of this book is an experience that makes a teacher wonder why he should have been deprived of it during his days of ignorance, for within its covers is a concentrate of suggestions for adding to the textbook content of mathematics instruction.

It lists sources of varied types of useful materials like books on mathematical subjects for pupils and teachers, plays, pictures, lantern slides, models, instruments, and the like. It offers suggestions for activities and projects like giving assemblies, conducting clubs, and arranging exhibits. Enough description and information is given about each item to enable the teacher to judge its use and value and to obtain the materials recommended.

Besides being checked and brought up to date, the book has been amplified by the addition of new sections and new material within sections. Among other additional topics whose importance has developed in the past ten years are health and safety and workbooks. Fortunately, an index has been added.

E.W.

Elements of the Theory of Integers. By Joseph Bowden. Published by Joseph Bowden, Garden City, New York, 1931. x+268 pp. Price, $3.00.

According to the author this book resulted from "a desire to put the elementary theory of numbers in logical form, starting from the three fundamental ideas of number, equality, and sum, with their axioms, building up a system of theorems on these fundamental ideas, and then, by as natural a process as possible, introducing the derived ideas of greater, less, difference, integer, product, quotient, and so forth."

This edition is a revision of an earlier edition where former errors are corrected, certain slight changes made in the text, and the chapter on congruence largely rewritten.

The chapters of the revised edition are as follows:

1. Fundamental Ideas. Axioms. Definitions. Number. Equality. Addition.
2. Subtraction. Greater. Less. Difference.
3. Positive Integers. Zero. Negative Integers.
4. Multiplication.
5. Numerical Value.
6. Division. Divisibility. Factors. Quotient.
7. Factors.
8. Greatest Common Factor.
9. Least Common Multiple.
10. Congruence.

Although the book has been published for some time, many readers of this magazine will doubtless not yet be familiar with it.

If the book is to be used as a text it would seem advisable to furnish a supplementary list of exercises, but perhaps the author has considered this matter and decided to omit any exercise material.

W.D.R.

Special Topics in Theoretical Arithmetic. By Joseph Bowden. Published by Joseph Bowden, Garden City, New York, 1936. xi+217 pp. Price, $2.50.

This book is intended by the author as a sequel to his *Elements of the Theory of Integers,* although it is independent of it. In this work he has incorporated a number of his discoveries, said to be published therein for the first time, some of which he has indicated by their dates of discovery. He says, however, that some of his discoveries may have been known before by others, and possibly in better form.

The book contains numerous items under the following chapter headings:

1. Series and Mathematical Induction.
2. Scales of Notation.
3. Congruence.
4. Indeterminate Equations of the First Degree in Two Unknowns.
5. Mathematical Recreations.

The author uses a system of simplified spelling which may or may not be acceptable, according to the person who happens to be using the work.

Teachers who are interested in these topics will wish to examine the books and compare them with similar treatises intended for immature though advanced students of mathematics.

W.D.R.